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BEAMFORMING COORDINADO PARA SISTEMAS MIMO-OFDM MULTIUSUARIO

TESIS

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DOCTOR EN CIENCIAS

Presenta:

LEONEL SORIANO EQUIGUA

Ensenada, Baja California, México, Noviembre de 2011

RESUMEN de la tesis de **LEONEL SORIANO EQUIGUA**, presentada como requisito parcial para la obtención del grado de DOCTOR EN CIENCIAS en ELECTRÓNICA Y TELECOMUNICACIONES con orientación en TELECOMUNICACIONES.
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BEAMFORMING COORDINADO PARA SISTEMAS MIMO-OFDM MULTIUSUARIO

Resumen aprobado por:

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Los sistemas de comunicación inalámbricos con múltiples antenas tanto en el transmisor como en el receptor (MIMO), son ampliamente reconocidos como la tecnología clave para alcanzar altas tasas de transmisión. Los sistemas MIMO pueden incrementar la capacidad del canal y la robustez del enlace de comunicaciones inalámbrico explotando la multidimensionalidad del canal creada por el número de antenas empleadas. Los sistemas MIMO multiusuario (MU-MIMO) combinan la alta capacidad alcanzable por los sistemas MIMO con los beneficios del acceso múltiple por división espacial. Beamforming coordinado (CBF) es una familia de algoritmos para MU-MIMO que alcanza altas capacidades del sistema como resultado de optimizar conjuntamente los vectores de beamforming en el transmisor y los vectores de combinación en los receptores.

Esta tesis presenta algoritmos avanzados de CBF para sistemas MU-MIMO y MU-MIMO-OFDM aplicando la técnica de feedforward limitada.

Como una primer contribución, se proponen tres métodos para calcular conjuntamente los vectores de beamforming y de combinación, los algoritmos usan información cuantizada de los beamformers para obtener los combinadores correspondientes, maximizando la relación señal-a-interferencia-más-ruido. Dos de los métodos propuestos superan en desempeño a los existentes en la literatura. El tercer método reduce el sobre encabezado en el enlace de feedforward con una ligera degradación en el desempeño. Nuestras tres propuestas están basadas en una cancelación perfecta de interferencia en el transmisor antes de calcular los beamformers cuantizados.

Nuestra segunda contribución es la optimización del algoritmo CBF iterativo basado en descomposición en valores singulares (CBF-SVD), nuestro algoritmo mejorado alcanza un mejor desempeño en términos de tasa de bit errónea con una cantidad menor de iteraciones.

Finalmente, se analiza la aplicación de algoritmos CBF en canales selectivos en frecuencia aplicando multiplexión por división de frecuencia ortogonal (OFDM). Como resultado, se proponen cuatro nuevos algoritmos para implementar CBF en sistemas MU-MIMO-OFDM. Se explota la correlación para reducir el sobre encabezado en el enlace de feedforward, mientras se maximiza la capacidad del sistema.

Palabras Clave: MU-MIMO, MIMO-OFDM, beamforming coordinado, feedforward limitada.

ABSTRACT of the thesis presented by **LEONEL SORIANO EQUIGUA**, in partial fulfillment of the requirements for the degree of DOCTOR OF SCIENCES in ELECTRONICS AND TELECOMMUNICATIONS with orientation in TELECOMMUNICATIONS. Ensenada, Baja California, November 2011.

COORDINATED BEAMFORMING FOR MULTIUSER MIMO-OFDM SYSTEMS

Multiple-input multiple-output (MIMO) communication, is widely acknowledged as the key technology for achieving high data rates in wireless systems. MIMO systems can increase the channel capacity and link robustness of wireless communication by exploiting the multi-dimensional wireless channel created by multiple transmit and receive antennas. Multiuser MIMO (MU-MIMO) systems combine the high capacity achievable with MIMO systems with the benefits of space division multiple access. Coordinated beamforming (CBF) is a family of algorithms for MU-MIMO that achieves high sum rates as a result of jointly optimizing both transmit beamforming and receive combining vectors.

This dissertation presents enhanced CBF algorithms for MU-MIMO and MU-MIMO-OFDM systems with limited feedforward.

As a first contribution, three methods to jointly calculate beamforming and combining vectors are proposed, the algorithms use quantized information of the beamformers to get the corresponding combiners, maximizing the signal-to-interference-plus-noise-ratio. Two of the proposed methods exceed in performance those already published. The third method reduces the feedforward overhead with a light degradation in performance. Our three proposals are based on a perfect interference cancelation at the transmitter before calculating the quantized beamformers.

Our second contribution is the optimization of the existing iterative CBF algorithm based on singular value decomposition (CBF-SVD), our improved algorithm offers a better performance in terms of bit error rate with a lower number of iterations.

Finally, the application of CBF algorithms in frequency selective channels is also analyzed by applying orthogonal frequency-division multiplexing (OFDM). As a result, four new algorithms are proposed for implementing CBF in MU-MIMO-OFDM systems. The correlation between subcarriers is exploited for reducing the feedforward overhead, while maximizing the sum rate

Keywords: MU-MIMO, MIMO-OFDM, coordinated beamforming, limited feedforward.

To my wife, Laura,

and

my little kids, Alex and Karime

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Chapter I

Introduction

I.1 Preliminaries

The use of antenna arrays at both the transmitter and the receiver has received significant attention as a promising method to provide diversity and/or multiplexing gain over wireless links. *Multiple antennas create extra dimensions in the signal space which can be used in different ways.* The receiver can be provided with replicas of the same data to increase the reliability of signal transmission which results in spatial diversity gain.

Multiple-Input Multiple-Output (MIMO) communication, is widely acknowledged as the key technology for achieving high data rates in wireless communication systems. MIMO systems can increase the channel capacity and link robustness of wireless communication by exploiting the multi-dimensional wireless channel created by multiple transmit and receive antennas (Telatar, 1999), (Foshini and Gans, 1998), (Gesbert *et al.*, 2003).

MIMO techniques were first investigated in a point-to-point or *single-user communication* link. In a MIMO single-user system with M_t transmit and M_r receive antennas, a diversity order of $M_t \times M_r$ can be provided for the system. Also, if the channel is perfectly known at the receiver, capacity scales linearly with $min(M_t, M_r)$ relative to a system with just one transmit and one receive antenna. A MIMO system is thus able to provide improved power and bandwidth efficiencies, at the cost of setting up additional antennas.

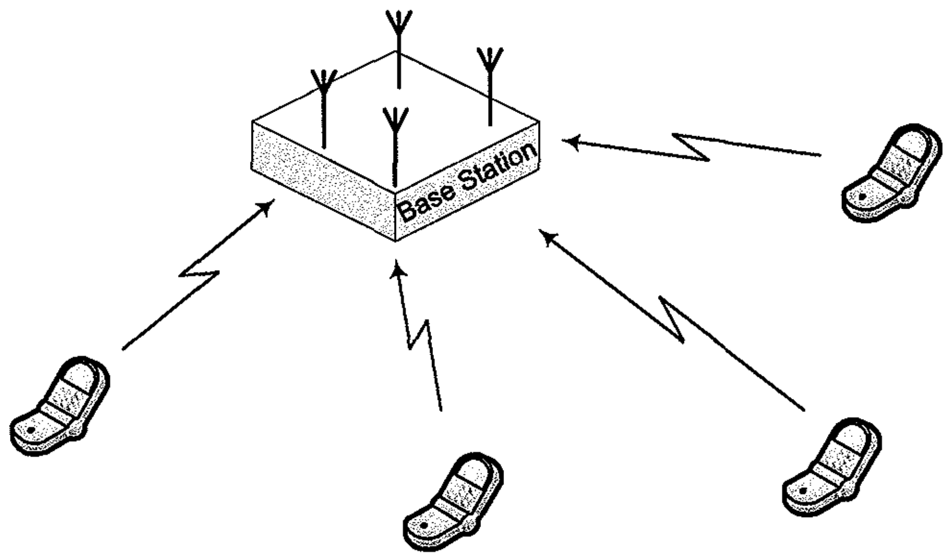

Figure 1. Multiple access channel.

Multiuser MIMO (MU-MIMO) systems combine the high capacity achievable with MIMO systems with the benefits of Space Division Multiple Access (SDMA). Such systems consider a Base Station (BS) that transmits to multiple mobile stations Mobile Station (MS)s simultaneously over the same frequency band, with a substantial increase in the channel capacity compared to other multi-access schemes (Spencer *et al.*, 2004).

There are two basic multiuser MIMO channel models: the MIMO Multiple-Access Channel (MAC) and the MIMO Broadcast Channel (BC). In MIMO MAC, a number of users share a common communication channel to transmit their individual signals to a receiver. Such a system is shown in Figure 1. In the uplink of a mobile cellular communication system, the users are the mobile transmitters in any particular cell and the receiver is the base station of that cell.

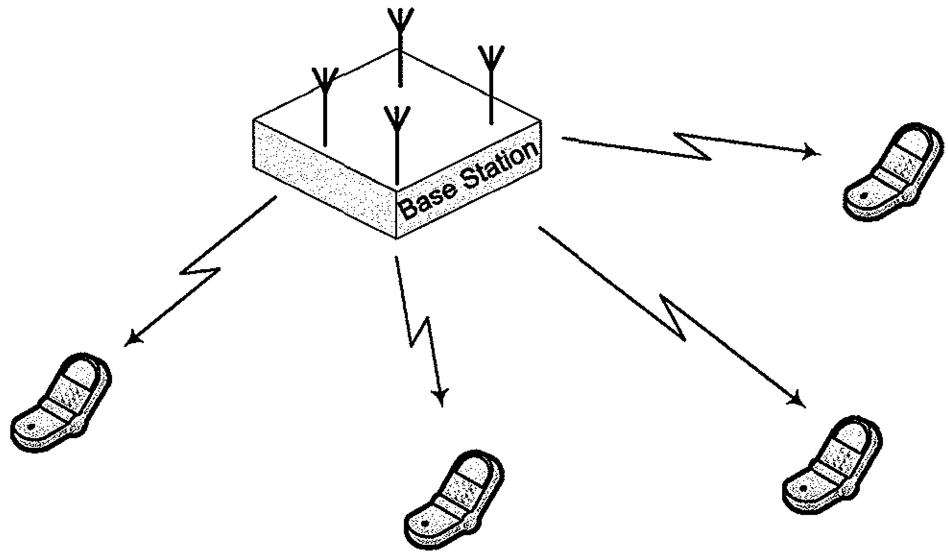

Figure 2. Broadcast channel.

In MIMO BC, a transmitter sends information to multiple receivers as shown in Figure 2. In the downlink of a mobile cellular communication system, the transmitter is the BS and the receivers are the MSs.

The broadcast channel and multiple access channel can be separated via Time-Division Duplex (TDD) or Frequency-Division Duplex (FDD). *In both FDD and TDD systems, the* knowledge of Channel State Information (CSI) has been mandatory to make use of a variety of channel adaptive techniques. A practical technique to obtain CSI in FDD systems is limited feedback, a methodology for obtaining and exploiting CSI at the transmitter. It uses a low rate feedback control channel to convey channel's quantized information from the receiver to the transmitter (see figure 3). In single-user MIMO case that information can be a quantized transmit beamforming vector while in MU-MIMO (with FDD) can be a quantized version of the user's channel or quantized versions of SNR or rate of each user (Love *et al.*, 2008).

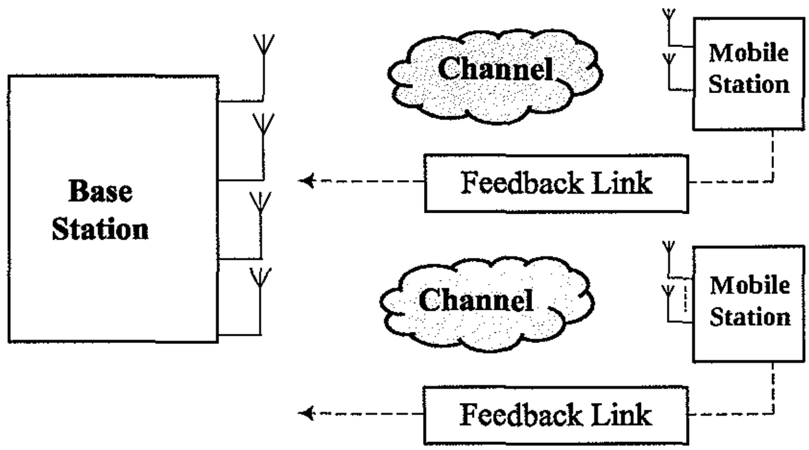

Figure 3. MU-MIMO system with limited feedback.

TDD is one of the modes included in the cellular 3GPP Long Term Evolution (LTE) standard, and it is best applicable to urban, local area or office deployments, where the transmit powers, mobile speeds, and the channel propagation delays are relatively low.

In TDD systems, the BS can get all the users channel information if the receivers have already done multiple access; this is possible thanks to reciprocity, where the downlink transmit channel can be inferred based on an estimate of the uplink received channel, when the time between transmission and reception is smaller than the coherence time of the channel, at the cost of a careful system design and calibration.

The limited feedback proposal has been recently extended to TDD MU-MIMO systems (Chae *et al.*, 2008), where the BS can estimate the CSI of all users via reciprocity and implements a control channel to inform each user about the correct combining vector in order to achieve a higher sum rate. This technique is named limited feedforward because the information is sent from the BS to the receivers (see figure 4). As in limited feedback, in limited feedforward a set of codebooks is used to quantize the beamforming vectors and reduce the overhead in the low rate link.

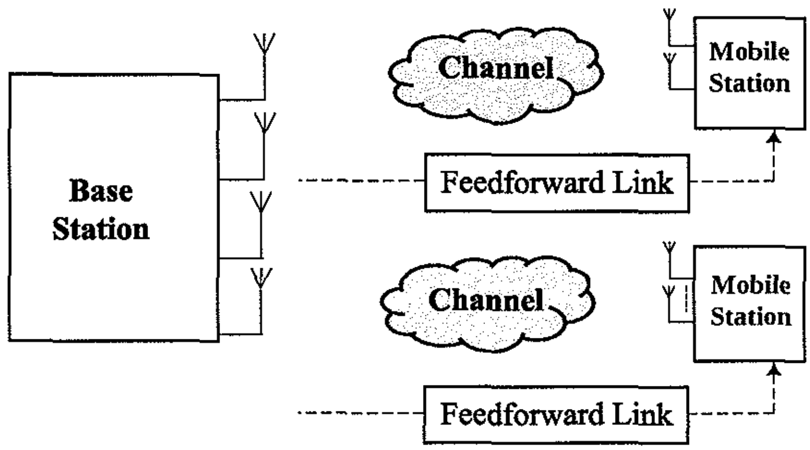

Figure 4. MU-MIMO system with limited feedforward.

MIMO-OFDM converts a broadband channel into a set of parallel narrow-band MIMO channels, named sub-channels, by appending cyclic prefix to each data symbol block and applying Discrete Fourier Transform (DFT)/Inverse Discrete Fourier Transform (IDFT). Limited feedforward for MU-MIMO-OFDM can be naturally extended from the narrow-band designs by performing CSI feedforward for each sub-carrier. The drawback of this approach is that the total feedforward increases linearly with the number of sub-channels. Consequently, this intuitive approach potentially causes a feedforward bottleneck since the number of sub-channels can be a few thousands in practical systems. More efficient broadband feedforward techniques can be designed by observing the correlation in CSI for neighboring sub-channels and consequently in their corresponding beamformers.

In this thesis we deal with MU-MIMO and MU-MIMO-OFDM systems with TDD and limited feedforward.

I.2 Problem Statement and Motivation

In new generation wireless networks like Worldwide Interoperability for Microwave Access (WiMAX) (Relay Task Group, 2011) and 3GPP LTE (Duplicy *et al.*, 2011), multiuser MIMO (MU-MIMO) is a reality. Therefore, recently researchers have been attracted to investigate the impact and implications of using MIMO systems in multiuser environments.

MU-MIMO need to be studied in order to provide wireless communication systems to obtain higher sum rates that new applications are demanding. In this sense, Dirty Paper Coding (DPC) was proposed as the optimal strategy given by information theory for the MIMO broadcast channel. It achieves the maximum sum rate, however it is difficult to implement due to its high complexity (Zhang *et al.*, 2009).

Several practical near-DPC techniques based on the concept of precoding have been proposed with different tradeoffs between complexity and performance. One of the simplest approaches for multiuser precoding is to premultiply the transmitted signal by a suitably normalized inverse of the multiuser channel matrix through Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) (Peel *et al.*, 2005). Both ZF and MMSE have the advantage of being relatively easy to implement, but require one receive antenna per user. Other proposal is called block diagonalization, that enforces a zero interference property at each user but requires the number of receive antennas to be equal to the number of data streams for each user.

Coordinated Beamforming (CBF), a generalization of block diagonalization, provides high sum rates for downlink communication in the MU-MIMO channel and does not impose any restriction on the number of receive antennas subject to send one stream of data per user. The benefits of CBF are a result of jointly optimizing both transmit beamforming and receive combining vectors (Farhang-Boroujeny *et al.*, 2003), (Choi and Murch, 2004), and (Pan *et al.*, 2004).

The basic concept of this kind of schemes is to use a group of transmit beamforming and receive combining vectors that ensures zero inter-user interference and maximizes the sum rate of the system, by taking advantage of the CSI.

In TDD systems, the joint optimization can be performed at the BS if transmit CSI is available. As was before mentioned, thanks to reciprocity, the downlink transmit channel can be inferred based on an estimate of the uplink received channel when the time between transmission and reception is smaller than the coherence time of the channel. CBF thus computes both the transmit beamforming vectors and receive combining vectors at the base station. Unfortunately, the receive combining vectors in CBF can not be computed based on channel state information available at each receiver in systems that employ a common pilot channel for training, essentially where the training data is sent prior to beamforming. To solve this problem, a CBF with limited feedforward was proposed in (Chae *et al.*, 2006) and (Chae *et al.*, 2008), where the combining vectors are quantized at the base station and sent to the receivers through a limited feedforward control channel.

CBF with limited feedforward is a promising proposal that, together Orthogonal Frequency Division Multiplexing (OFDM), can be applied on new generation wireless MIMO communication systems such as 3GPP LTE.

I.3 Objectives

The main objectives of this thesis are:

Objective 1: Study the advances in CBF with limited feedforward in order to propose practical CBF methods for improving the capacity of the MU-MIMO wireless systems with reduced complexity.

Objective 2: Analyze the application of CBF algorithms in MU-MIMO-OFDM systems and propose methods to reduce the overhead in the feedforward link subject to the system's sum rate maximization.

I.4 Contributions of this Thesis

This thesis is concerned with the investigation of CBF for MU-MIMO systems with limited feedforward and its extension to MIMO-OFDM systems. The original contributions of this work are listed as follows.

1. Three novel non-iterative algorithms for CBF with limited feedforward. The algorithms use quantized information of the beamformers to get the corresponding combiners, maximizing the Signal-to-Interference-plus-Noise-Ratio (SINR). Two of the proposed methods exceed in performance those already published. The third method reduces the feedforward overhead with a light degradation in performance. Our three proposals are based on a perfect interference cancellation at the transmitter before calculating the quantized beamformers.

2. An improved version of the iterative CBF algorithm based on Singular Value Decomposition (SVD) reported in (Chae *et al.*, 2006). Our improved algorithm performs better in the sense of that it is more efficient in terms of convergence and bit error rate performance.

3. Four CBF methods for MU-MIMO-OFDM. Three of our proposal choosing the best quantized beamforming vector that represents a subcarrier group, for coordinated beamforming in the downlink of MU-MIMO-OFDM systems. The interpolation based proposal chooses the firs and last beamformers of the cluster and a phase parameter to reconstruct the rest of the beamformers group, in order to maximize the sum rate of the subcarrier group. In all algorithms the correlation between subcarriers is exploited for reducing the feedforward overhead, while maximizing the sum rate. The algorithm with best performance is advanced clustering.

As a result of our contributions the following papers were accepted to be published in recognized journals and conferences.

- Soriano-Equigua, L., Sánchez-García, J., Flores-Troncos, J., and Heath, R. W. (2011). Non-Iterative Coordinated Beamforming for Multiuser MIMO Systems with Limited Feedforward. *IEEE Signal Processing Letters*. **18**(12):701-704.

- Soriano-Equigua, L., Sánchez-García, J., Chae, C.-B., and Heath, R. W. (2011). Improved Iterative Coordinated Beamforming Based on Singular Value Decomposition for Multiuser Mimo Systems With Limited Feedforward. *Journal of Applied Research and Technology*. **9**(3):342-354.

- Soriano-Equigua, L., Sánchez-García, J., Chae, C.-B., and Heath, R. W. (2011). Overhead Reduction in Coordinated Beamforming for Multiuser MIMO-OFDM Systems with Limited Feedforward. *IEICE Transactions on Communications*. **E94-B**(11):3168-3171.

- Soriano-Equigua, L., Sánchez-García, J., Flores-Troncoso, J., and Alvarez-Flores, J. L. (2010). Técnicas de reducción de overhead en sistemas de comunicaciones inalámbricos MIMO-OFDM con feedback limitado. *Procedings of IEEE Reunión Internacional de Otoño de Comunicaciones, Computación, Electrónica, Automatización, Robótica y Exposición Industrial*, Acapulco, Gro., México, November 28 - December 4, 2010.

- Soriano-Equigua, L., Sánchez-García, J., Chae, C.-B., and Heath, R. W. (2010). Quantized Coordinated Beamforming with Phase Rotation for Bit Error Rate Improvement. *IEEE Communication Theory Workshop*, Cancún, Q. Roo, México, May 5-7, 2010.

I.5 Thesis Outline

The organization of this thesis is as follows.

In chapter II, we provide a general overview of a CBF for MU-MIMO with limited feedforward system. We describe the system elements that also will be considered for the chapter III. After that, we describe our proposed algorithms and analyze their performance to finally remark some conclusions.

In chapter III, we review the already published iterative CBF algorithm based on SVD, then we analyze their weaknesses in order to propose its optimization in terms of number of iterations and bit error rate performance. The simulation results are analyzed and a brief set of conclusions are given.

In chapter IV, we study the main algorithms that have been proposed to reduce the overhead in MIMO-OFDM systems with limited feedback. These algorithms are based in the instantaneous channel state. Their performance in terms on bit error rate is analyzed in the single user case. As a consequence of the results, we adopt clustering and interpolated beamforming techniques to propose algorithms to reduce the overhead in CBF for MU-MIMO-OFDM.

In chapter V, four strategies to reduce the feedforward overhead in a multiuser MIMO-OFDM system subject to maximizing the sum rate are proposed. We start the chapter providing a short review of the iterative CBF algorithm based on matrix inversion that was used to calculate the beamformers and combiners. Next, our algorithms are described and its performance is analyzed under different quantization settings, different channel profiles, and different conditions of path loss. At the end some conclusions are remarked.

Finally, chapter VI contains some concluding remarks and discussion on future research ideas on this topic.

I.6 Notation

In this thesis we use uppercase and lowercase boldface letters to denote matrices and vectors, and the operations on scalars, vectors and matrices are denoted as follows:

$\boldsymbol{A}^T$	Transpose of $\boldsymbol{A}$
$\boldsymbol{A}^H$	Hermitian of $\boldsymbol{A}$
$\boldsymbol{A}^\dagger$	Pseudo inverse of $\boldsymbol{A}$
$\|\|\boldsymbol{A}\|\|$	2-norm of $\boldsymbol{A}$
cols($\boldsymbol{A}$)	columns of matrix $\boldsymbol{A}$

$\lvert\cdot\rvert$	The absolute value of $(\cdot)$
$\mathbb{E}[\cdot]$	Expectation
$\lceil\cdot\rceil$	Ceil operation
$\mathrm{svd}(\boldsymbol{A})$	Singular value decomposition of $\boldsymbol{A}$
$< \boldsymbol{a}, \boldsymbol{b} >$	Dot product between $\boldsymbol{a}$ and $\boldsymbol{b}$
$\boldsymbol{A}[:, u]$	The u-th column of the matrix $\boldsymbol{A}$

Chapter II

Non-iterative Coordinated Beamforming for MU-MIMO Systems with Limited Feedforward

II.1 Introduction

In this chapter we present the results published in (Soriano-Equigua *et al.*, 2011c), where we propose three CBF algorithms for MU-MIMO. We first analyzed the methods proposed in (Chae *et al.*, 2008), where a full search and two low complexity CBF algorithms were proposed. The full search algorithm, named as *joint receiver quantization*, performs better in terms of sum rate at the cost of a major complexity, and its performance approaches the sum capacity. The best suboptimal algorithm (iteration-based independent quantization) has a marginal gap in sum rate performance with respect to joint receiver quantization with less and variable complexity due to its iterative nature.

One of the most important differences of this work compared to (Chae *et al.*, 2008) is how it handles the quantized beamforming vectors. In (Chae *et al.*, 2008), the transmitter leaves the interference cancellation as part of the quantized beamformer optimization. In this work, the interference cancellation part is done perfectly in the transmitter, before calculating the quantized beamformers. By taking advantage of this fact, we propose an improved full search CBF algorithm that achieves a higher sum rate than joint receiver quantization.

Furthermore, two suboptimal non-iterative proposals are presented to reduce the complexity. Our three proposals were tested, through simulations, in a 4x4 MIMO system.

II.2 System Model

Consider the downlink transmission of a multiuser MIMO system with N_t transmit antennas, N_r receive antennas, and U users as illustrated in figure 5. A complex symbol x_u transmitted by the uth ($1 \leq u \leq U$) user is multiplied by a transmit beamforming vector $\boldsymbol{f}_u$, then added to the beamformed data that belong to remaining users. The base station launches the resulting signal into the propagation environment. The necessary information to calculate the combiners is sent to each user through a limited feedforward link, which is assumed with zero delay.

The signal y_u received by the uth user after processing with the combining vector $\boldsymbol{w}_u$ is given by

$$y_u = \boldsymbol{w}_u^H \left(\boldsymbol{H}_u \sum_{\ell=1}^{K} \boldsymbol{f}_\ell x_\ell + \boldsymbol{v}_u \right), \tag{1}$$

where $\boldsymbol{v}_u$ is a vector of independent identically distributed (i.i.d.) complex zero-mean Gaussian noise with variance σ_v^2, $\boldsymbol{f}_u$ and $\boldsymbol{w}_u$ are the transmit beamforming vectors and receive combining vectors respectively, calculated by using coordinated beamforming. As $\boldsymbol{f}_u$ and $\boldsymbol{w}_u$ are unitary vectors the noise is not amplified at the receiver when the combining vector is applied. $\boldsymbol{H}_u$ is the channel at the uth MS represented by a matrix of size $N_r \times N_t$.

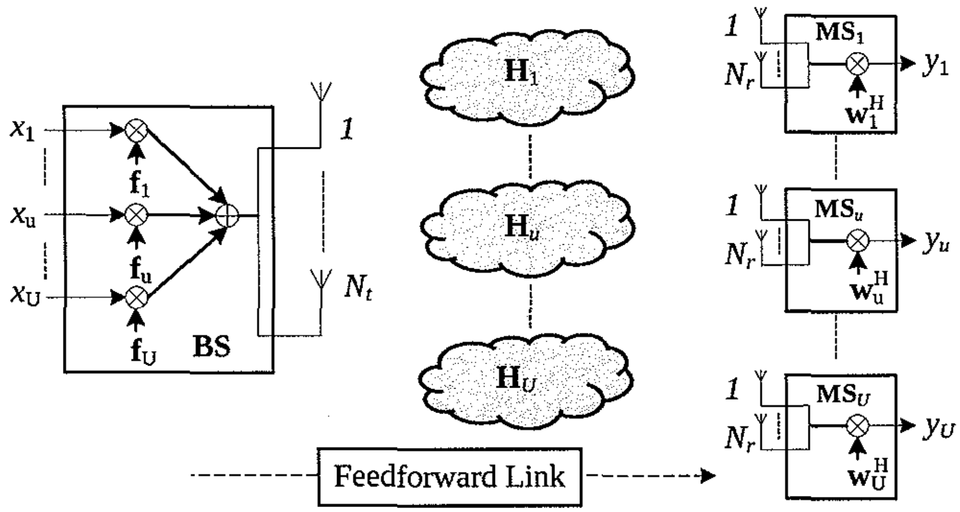

Figure 5. Coordinated beamforming for MU-MIMO system model with limited feedforward.

The combining vector is computed at the uth receiver by applying Maximum Ratio Combining (MRC), $\boldsymbol{w}_u = \boldsymbol{H}_u\boldsymbol{c}_{i_u}/||\boldsymbol{H}_u\boldsymbol{c}_{i_u}||$, where $\boldsymbol{c}_{i_u}$ is the code corresponding to the i_uth codebook index sent by the BS. The transmit beamforming vector of the uth user, $\boldsymbol{f}_u$, is calculated in order to cancel the interference to the reminder rest of the receivers, that is, $\boldsymbol{w}_l^H\boldsymbol{H}_l\boldsymbol{f}_u = 0$ (for $l \neq u$).

II.3 CBF with Limited Feedforward Review

In this section we review the Joint Receive Quantization (JRQ) algorithm, presented in (Chae *et al.*, 2008). Is assumed in this algorithm that there is a codebook $C = \{c_1, c_2, \ldots, c_{2^b}\}$ that is shared between the BS and the mobile stations, where b is the number of bits of the codebook. JRQ is a full search based algorithm that can be summarized as follows.

1. Initialization. The BS computes the matched channel matrix for each user, defined for the u-th user as $\boldsymbol{R}_u = \boldsymbol{H}_u^H \boldsymbol{H}_u$.

2. Calculate the effective channel matrix. The MIMO channel of each user can be simplified in an effective channel derived from the application of a combiner at the receiver (Jindal, 2008), (Sánchez-García *et al.*, 2009). Assuming that the u-th receiver uses a i-th code $\boldsymbol{c}_{i_u}$ to compute its combiner as $\boldsymbol{w}_u = \boldsymbol{H}_u \boldsymbol{c}_{i_u}$, its effective channel is calculated as

$$\boldsymbol{h}_{eff,u} = \boldsymbol{w}_u^H \boldsymbol{H}_u = (\boldsymbol{H}_u \boldsymbol{c}_{i_u})^H \boldsymbol{H} = \boldsymbol{c}_{i_u}^H \boldsymbol{H}_u^H \boldsymbol{H}_u = \boldsymbol{c}_{i_u}^H \boldsymbol{R}_u. \tag{2}$$

   Then, the effective channel matrix is computed as

$$\boldsymbol{H}_{\text{eff}}(i_1, ..., i_u, ..., i_U) = \left[\left(\boldsymbol{c}_{i_1}^H \boldsymbol{R}_1\right)^T \cdots \left(\boldsymbol{c}_{i_u}^H \boldsymbol{R}_u\right)^T \cdots \left(\boldsymbol{c}_{i_U}^H \boldsymbol{R}_U\right)^T \right]^T. \tag{3}$$

3. Calculate the transmit beamformers. Based on the effective channel matrix $\boldsymbol{H}_{\text{eff}}$, the BS computes the transmit beamformers as follows:

$$[\boldsymbol{f}_1 \cdots \boldsymbol{f}_u \cdots \boldsymbol{f}_U] = \text{cols}\left(\frac{\boldsymbol{H}_{\text{eff}}^{\dagger}(i_1, ..., i_u, ..., i_U)}{\|\boldsymbol{H}_{\text{eff}}^{\dagger}(i_1, ..., i_u, ..., i_U)\|} \right). \tag{4}$$

4. Evaluate the achievable sum rate for each receiver. Using the computed transmit beamforming vector $\boldsymbol{f}_u$ and the quantized code $\boldsymbol{c}_{i_u}$, the transmitter computes de achievable sum rate for each receiver as

$$R_k(i_1, i_2, \ldots, i_U) = \log_2 \left(1 + \frac{|\boldsymbol{c}_{i_u} \boldsymbol{R}_u \boldsymbol{f}_{i_u}|^2}{\boldsymbol{c}_{i_u}^H \sum_{\ell=i_1, \ell \neq i_k}^{i_u} \boldsymbol{R}_u \boldsymbol{c}_\ell \boldsymbol{c}_\ell^H \boldsymbol{R}_u \boldsymbol{c}_{i_u} + \boldsymbol{c}_{i_u}^H \boldsymbol{R}_u \boldsymbol{c}_{i_u} \sigma_u^2} \right). \tag{5}$$

5. Find de optimum codebook indexes. To find the codebook indexes for the U receivers the following maximization is solved

$$\left(\hat{i}_1, \hat{i}_2, \ldots, \hat{i}_U\right) = \underset{i_1, \ldots, i_2, \ldots, i_U}{\arg\max} \sum_{u=1}^{U} R_k(i_1, i_2, \ldots, i_U). \tag{6}$$

6. Cancel residual interference. With the codebook indexes $\left(\hat{i}_1, \hat{i}_2, \ldots, \hat{i}_U\right)$, the BS recomputes the effective channel matrix using (3), and also executes step 3 to calculate the final beamformers.

II.4 Proposed CBF Algorithms

In this section, we propose three new non-iterative algorithms for CBF with limited feedforward. The first algorithm is an improved version of the joint receiver quantization algorithm presented in (Chae *et al.*, 2008), while the second and third algorithms are suboptimal proposals that reduce the complexity of the new full search based algorithm.

II.4.1 Improved Joint Receiver Quantization

As in (Chae *et al.*, 2008), we consider a shared codebook for the BS and the receivers, so by taking into account all possible combinations of the codes for all users, we calculate steps 1, 2 and 3 of the algorithm described in section II.3.

The previously calculated beamformers impose zero interference when the uth receiver uses $\boldsymbol{w}_u = \boldsymbol{H}_u \boldsymbol{c}_{i_u} / \|\boldsymbol{H}_u \boldsymbol{c}_{i_u}\|$, where $u = 1 \ldots U$, as its combining vector. Based on this fact, we propose to find the codebook indexes that maximize the sum rate, that after interference cancelation is reduced to

$$(i_1, ..., i_u, ..., i_U) = \underset{i_1,...,i_u,...,i_U}{\arg\max} \sum_{u=1}^{U} \log_2 \left(1 + \frac{|\boldsymbol{c}_{i_u}^H \boldsymbol{R}_u \boldsymbol{f}_u|^2}{\sigma_u^2} \right). \tag{7}$$

By selecting the codebook indexes in this way, our algorithm computes $\left(2^b\right)^U$ pseudo inverse calculations, where b is the number of bits used for the index of the codebook, one pseudo inverse calculation less than joint receiver quantization in (Chae *et al.*, 2008) (due to the interference cancellation step is not necessary). Another advantage of our proposal is that equation number (7) has a complexity lower than that of equation (6) employed in (Chae *et al.*, 2008) to select the codebook indexes, because our proposal does not introduce any interference term in the expression to be optimized.

II.4.2 Suboptimal Quantization

Both joint receiver quantization and improved joint receiver quantization have fixed complexity, which is mainly determined by the codebook size, the number of users, and the number of transmit and receive antennas. In this section, we propose a suboptimal method that reduces the complexity of the full search CBF algorithms. Our algorithm can be summarized as follows.

1. At the initialization step, the users are sorted in descending order by considering the norm of its channel as ranking criteria.
2. Form a subgroup of effective channels for each user, the subgroup for the uth receiver is given by $\boldsymbol{h}_{\mathrm{eff},i_u} = \boldsymbol{c}_{i_u}^H \boldsymbol{R}_u$, where $i_u = 1, \ldots, 2^b$ is the codebook index for the uth receiver.
3. Select 2^b posible U-tuples of codes $(i_1, \ldots, i_u, \ldots, i_U)$ by taking as reference the subgroup $\boldsymbol{h}_{\mathrm{eff},i_1}$.
   (a) The first element of the first Utuple is initialized as $i_1 = 1$,
   (b) the i_uth element is selected such that

$$i_u = \arg\max_{i_u} \sum_{l=1}^{U} \log_2 \left(1 + \frac{|\boldsymbol{c}_{i_l}^H \boldsymbol{R}_l \boldsymbol{f}_l|^2}{\sigma^2} \right), \tag{8}$$

where the beamformers $\{f_l\}_{l=1}^{U}$ are calculated from the pseudo inverse of

$$\boldsymbol{H}_{\text{eff}}(i_1,\ldots,i_U) = \left[\boldsymbol{h}_{\text{eff},i_1}^T,\ldots,\boldsymbol{h}_{\text{eff},i_U}^T\right]^T. \tag{9}$$

(c) Repeat a) and b) for $i_1 = 2,\ldots,2^b$ until the 2^b Utuples are computed.

4. Select the Utuple that maximizes (7).

For $U > 2$, this algorithm computes $\left(2^b\right)^2$ pseudo inverses of matrices of size $2 \times U$ plus $\left(2^b\right)^2$ pseudo inverses of matrices of size $3 \times U$ and so on, up to $\left(2^b\right)^2$ pseudo inverses of matrices whose size is $U \times U$.

For the case $U = 2$, the suboptimal quantization behaves exactly as the improved joint receiver quantization, performing $\left(2^b\right)^2$ pseudo inverses of matrices of size 2×2.

II.4.3 Low Complexity Quantization

A reduced complexity version of the suboptimal quantization is possible by modifying the step 3 and eliminating step 4 of this algorithm. For the reduced version, in step 3-a the codebook index i_1 is selected as

$$i_1 = \arg\max_{i_1} \|\boldsymbol{H}_1 c_{i_1}\|. \tag{10}$$

The step 3-b remains unchanged, and step 3-c is eliminated.

This reduced complexity version decreases the necessary feedforward overhead by a factor of $(U-1)/U$, because the first user always apply the code that maximizes (10) and this code is known at the receiver. The number of pseudo inverse operations for this method is,

2^b pseudo inverses of matrices of size $2 \times U$ plus 2^b pseudo inverses of matrices of size $3 \times U$ and so on, up to 2^b pseudo inverses of matrices of size $U \times U$.

II.4.4 Comments about computational complexity

The overall computational complexity of our algorithms is mainly determined by the number of the effective channel matrix inversions. The computational complexity of matrix inversion of $\boldsymbol{H}_{\text{eff}}$ is $O((min(U, N_r))^2 log(min(U, N_r)))$ which is mentioned in (Chae *et al.*, 2008).

II.5 Numerical Performance Analysis

In this section, we compare the performance of the proposed quantized CBF algorithms with those existing in the literature. Simulations results are presented for a MU-MIMO system, where i.i.d. complex Gaussian channels are assumed for the users. In all simulations we assume that

(i) the channels between different transmit and receive antenna pairs are independent,

(ii) the BS obtains a perfect channel estimate,

(iii) each MS correctly estimates its own channel

(iv) all users share the same codebook (the codebooks were generated using techniques reported in (Love *et al.*, 2003)..

Figure 6 shows results for the CBF system with $N_r = N_t = U = 4$, with a four bits size codebook and SNR values between zero and 20 dBs. It can be observed that for improved joint receiver quantization and suboptimal quantization algorithms, a better sum rate performance is achieved, compared with joint receiver quantization as well as with iterative quantized CBF. Furthermore, our low complexity proposal significantly reduces the gap in terms of sum rate with respect to joint receiver quantization at 20 dBs of SNR, with a feed-forward reduction rate of $(U-1)/U$. Fig. 6 also shows that the sum rate of our algorithms increases linearly as the SNR value is increased.

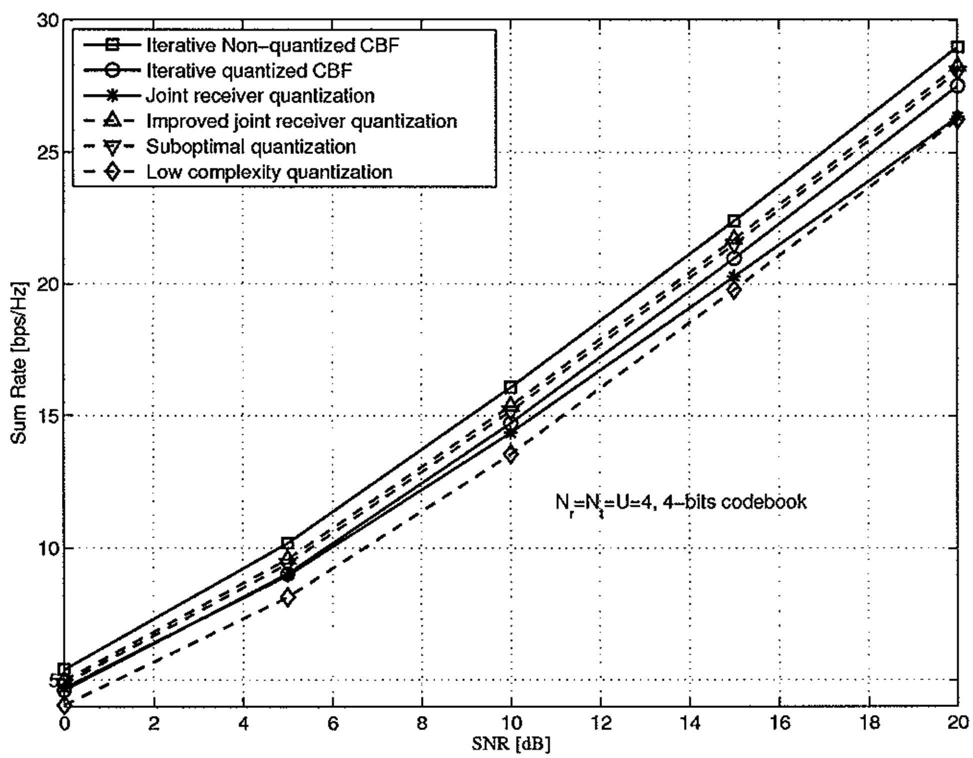

Figure 6. Sum rate versus SNR.

Figure 7 illustrates the performance of the proposed CBF algorithms for different code-book sizes. In this case the size of the code in bits varies between one and four while maintaining the system dimensions $N_r = N_t = U = 4$. The performance is analyzed for a SNR of 20 dBs. It can be observed that the sum rate increases an average of 20% when increasing the number of bits from one to two, while the increase in sum rate is just around 5% when increasing the number of bits from two to three. This last percent of increase persists for changing from three to four the number of bits used for code indices. It is worth to mention that, for the case of low complexity quantization, the performance is very similar to that of joint receiver quantization, with the advantage of a significative complexity reduction. The performance curve of iterative non-quantized CBF is included in figures 6 and 7 as a reference, to illustrate the loss in sum rate due to quantization.

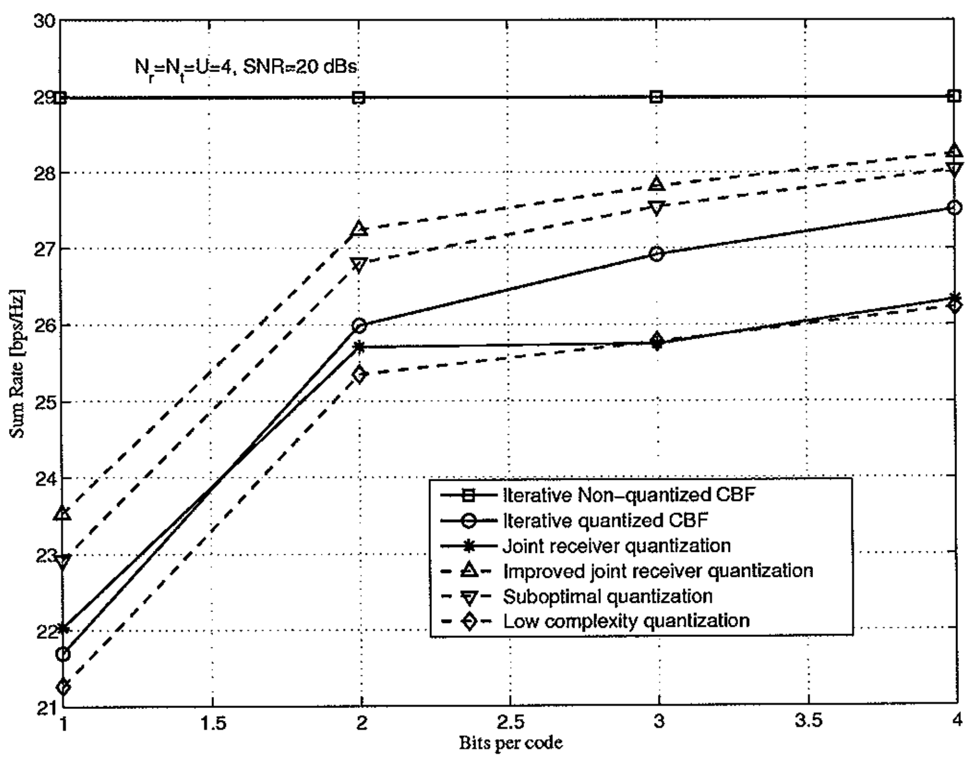

Figure 7. Sum rate versus bits/code.

II.6 Conclusion

In this chapter, three approaches to jointly calculate beamforming and combining vectors in a CBF MU-MIMO with limited feedforward were presented.

Unlike previous published methods, in this work the interference cancelation step is done perfectly in the transmitter, before calculating the quantized beamformers.

Simulation results have shown that two of the proposed algorithms, improved joint receiver quantization and suboptimal quantization, offer a performance better than joint receive quantization and iterative CBF, in terms of sum rate.

On the other hand, low complexity quantization reduces the overhead in the feedforward link by a factor of $(U-1)/U$ and is able to achieve almost the same sum rate that joint receive quantization, when $N_r = N_t = U = 4$ and the number of feedforward bits is greater than or equal to two.

Chapter III

Improved Iterative CBF Based on Singular Value Decomposition for MU-MIMO Systems with Limited Feedforward

III.1 Introduction

It has been shown in chapter II that CBF provides high sum rates for downlink communication in the MU-MIMO channel. One of the simplest approaches for CBF is the iterative CBF based on Singular Value Decomposition (CBF-SVD) proposed in (Chae *et al.*, 2006). This approach takes advantage of the full channel state information (CSI) at the transmitter to maximize the sum rate.

Several algorithms have been proposed for CBF with limited feedforward (see (Chae *et al.*, 2006) and (Chae *et al.*, 2008)); from the performance evaluation of these algorithms, it has been shown that the iterative algorithms achieve an excellent performance in terms of sum rate.

In this chapter we report the results published in (Soriano-Equigua *et al.*, 2010a) and (Soriano-Equigua *et al.*, 2011a), in this work we propose to optimize the CBF-SVD algorithm by improving the Bit Error Rate (BER) performance, reducing the necessary number of iterations, while achieving the same sum rate. Furthermore, the simulation results showed that the convergence of the algorithm was improved.

III.2 System Model and CBF-SVD Algorithm Review

In this section, we provide a short description of the multiuser MIMO system and give a review of the coordinated beamforming algorithm presented in (Chae *et al.*, 2006), which is taken as a basis for the development of this work.

Consider the downlink transmission of a multiuser MIMO system as is illustrated in section II.2.

The signal $\boldsymbol{y}_u$ received by the uth user after applying the combining vector $\boldsymbol{w}_u$ can be re-written as

$$y_u = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u x_u + \boldsymbol{w}_u^H \boldsymbol{H}_u \sum_{\ell=1}^{U} \boldsymbol{f}_\ell x_\ell + \boldsymbol{w}_u^H \boldsymbol{v}_u. \tag{11}$$

In equation (11), the first term represents the effective channel gain for the uth user, the second term shows the multi-user interference and the last term illustrates the vector noise multiplied by $\boldsymbol{w}_u$.

III.2.1 Coordinated Beamforming Algorithm Review

The iterative algorithm to compute the beamformers and combiners subject to sum rate maximization proposed in (Chae *et al.*, 2006), is shown in figure 8 and is summarized as follows:

1. Initialize the users' combining vectors to unit vectors. A good initialization is by setting $\boldsymbol{w}_u$ to the left singular vector that corresponds to the maximum singular value of $\boldsymbol{H}_u$, that is

$$[\boldsymbol{U} \quad \boldsymbol{D} \quad \boldsymbol{V}] = svd\,(\boldsymbol{H}_u) \tag{12}$$

$$\boldsymbol{w}_u = \boldsymbol{U}\,[:, 1] \tag{13}$$

where $\boldsymbol{U}\,[:, 1]$ represents the first column vector of $\mathbf{U}$.

2. Form the effective channel matrix as

$$\tilde{\boldsymbol{H}}_u = \left[\left(\boldsymbol{w}_1^H \boldsymbol{H}_1\right)^T \cdots \left(\boldsymbol{w}_{u-1}^H \boldsymbol{H}_{u-1}\right)^T \left(\boldsymbol{w}_{u+1}^H \boldsymbol{H}_{u+1}\right)^T \cdots \left(\boldsymbol{w}_U^H \boldsymbol{H}_U\right)^T\right]^T \tag{14}$$

3. Calculate the beamforming vector $\boldsymbol{f}_u$ such that it cancels the interference between users. The interference cancellation is done by calculating a vector orthogonal to the rows of $\tilde{\boldsymbol{H}}_u$; this is realized by calculating the SVD of $\tilde{\boldsymbol{H}}_u$ and taking as beamformer the right singular vector that corresponds to the singular value zero (the size of $\tilde{\boldsymbol{H}}_u$ is $(U-1) \times N_t$, if we consider $U = N_t$ the existence of one zero singular value is guaranteed). Then,

$$\left[\tilde{\boldsymbol{U}} \quad \tilde{\boldsymbol{D}} \quad \tilde{\boldsymbol{V}}\right] = svd\left(\tilde{\boldsymbol{H}}_u\right) \tag{15}$$

$$\boldsymbol{f}_u = \tilde{\boldsymbol{V}}\,[:, N_t] \tag{16}$$

where $\tilde{\boldsymbol{V}}\,[:, N_t]$ represents the last column vector of $\tilde{\boldsymbol{V}}$ (in SVD the right singular vectors are sorted such that the first column vector corresponds to the larger singular value and the last corresponds to the smaller singular value, in this case equal to zero).

4. Check the stopping criteria. The algorithm stops when the difference between the previous and current beamformers is small enough.

$$\|\boldsymbol{f}_{u,i} - \boldsymbol{f}_{u,i-1}\| < \epsilon \tag{17}$$

where i and $i-1$ represent the actual and previous iteration respectively.

5. Otherwise update the combining vectors using MRC

$$\boldsymbol{w}_u = \boldsymbol{H}_u \boldsymbol{f}_u \tag{18}$$

and go to step 2.

6. Quantize the beamformers. The beamformers are complex vectors that can take an infinite number of values; to send the index of the code that represent the current beamformer value, they need to be quantized. To find the quantized beamformers $\hat{\boldsymbol{f}}_u$ ($1 \leq u \leq U$), the transmitter selects the codebook index corresponding to the code that maximizes the signal-to-interference-plus-noise ratio ($SINR$)

$$\hat{\boldsymbol{f}}_u = \underset{\boldsymbol{c}_i \in C}{\arg\max} \quad \mathsf{SINR}_u(\boldsymbol{c}_i) \tag{19}$$

where

$$\mathsf{SINR}_u(\boldsymbol{c}_i) = \frac{\left|\boldsymbol{c}_i^H \boldsymbol{R}_u \boldsymbol{f}_u\right|^2}{\boldsymbol{c}_i^H \left(\sum_{l=1, l \neq u}^{U} \boldsymbol{R}_u \boldsymbol{f}_l \boldsymbol{f}_l^H \boldsymbol{R}_u \right) \boldsymbol{c}_i + \boldsymbol{c}_i^H \boldsymbol{R}_u \boldsymbol{c}_i \sigma^2}, \tag{20}$$

$\boldsymbol{R}_u = \boldsymbol{H}_u^H \boldsymbol{H}_u$ is the matched channel matrix and C is a Grassmannian codebook (Love *et al.*, 2003).

7. Cancel residual interference. After quantization, the BS updates the combiners, and executes steps two and three again to mitigate residual interference due to the quantization operation; we denote the final beamformers as $\{\tilde{\boldsymbol{f}}_u\}_{u=1}^{U}$.

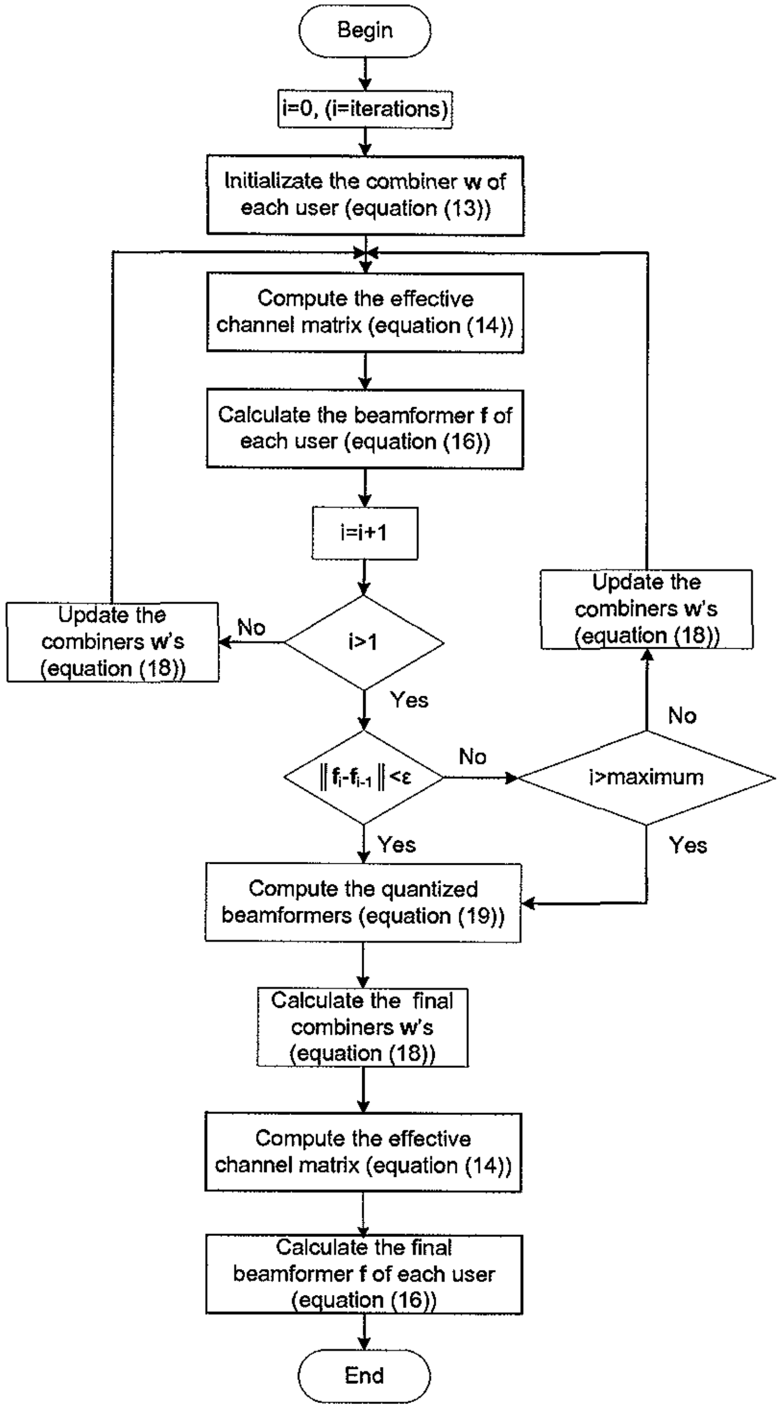

Figure 8. Flow diagram of the CBF-SVD algorithm.

III.3 Improved CBF-SVD Algorithm

In the previous section the CBF-SVD approach was reviewed, which is a transmitter based iterative technique for multiuser beamforming that improves the SINR in the system. After each iteration and quantization, the beamforming vectors are optimized in terms of sum rate. In this section, we show a method to improve the system BER performance by selecting the proper beamforming vectors. The optimization objective is to minimize the BER, which is equivalent to minimize the phase noise derived from applying the resulting beamformers at the transmitter and the combiners at the receivers.

III.3.1 Analysis of the Original Algorithm

Essentially, the algorithm described in subsection III.2.1 imposes zero interference on each iteration and evaluates the square root of the error in the obtained beamformers to determine the convergence of the algorithm. The beamforming vectors are selected to cancel the interference after the combining vectors are calculated. To illustrate the effects of the calculation of the beamformers in this way, we analyze the case of $N_t = N_r = U = 4$. Consider the ith iteration, the matrix $\tilde{\boldsymbol{H}}_u$ (with $u = 2$) is conformed as

$$\tilde{\boldsymbol{H}}_2 = \begin{bmatrix} \tilde{h}_{1,1} & \tilde{h}_{1,2} & \tilde{h}_{1,3} & \tilde{h}_{1,4} \\ \tilde{h}_{3,1} & \tilde{h}_{3,2} & \tilde{h}_{3,3} & \tilde{h}_{3,4} \\ \tilde{h}_{4,1} & \tilde{h}_{4,2} & \tilde{h}_{4,3} & \tilde{h}_{4,4} \end{bmatrix},$$

where $[\ \tilde{h}_{u,1}\ \ \tilde{h}_{u,2}\ \ \tilde{h}_{u,3}\ \ \tilde{h}_{u,4}\ ] = \boldsymbol{w}_u^H \boldsymbol{H}_u$. To cancel interference, the beamforming vector that belongs to the uth user ($u = 2$) satisfies

$$\begin{bmatrix} \tilde{h}_{1,1} & \tilde{h}_{1,2} & \tilde{h}_{1,3} & \tilde{h}_{1,4} \\ \tilde{h}_{3,1} & \tilde{h}_{3,2} & \tilde{h}_{3,3} & \tilde{h}_{3,4} \\ \tilde{h}_{4,1} & \tilde{h}_{4,2} & \tilde{h}_{4,3} & \tilde{h}_{4,4} \end{bmatrix} \begin{bmatrix} f_{2,1} \\ f_{2,2} \\ f_{2,3} \\ f_{2,4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

this is achieved by setting the beamforming vector as the right singular vector that corresponds to the singular value zero. As a consequence, equation (11) becomes

$$y_u = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u x_u + \boldsymbol{w}_u^H \boldsymbol{v}_u. \tag{21}$$

Without interference the SINR is reduced to

$$\mathsf{SINR}_u = \frac{\left|\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u\right|^2}{\sigma^2}, \tag{22}$$

where $\left|\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u\right|^2$ is the channel effective gain.

It is important to mention that to correctly decoding the transmitted symbol x_u, the product $\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u$ must be real and positive, otherwise the received symbol y_u will present phase noise and the BER performance will be degraded. The algorithm explained in subsection III.2.1 ensures the interference cancelation, but the result of $\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u$ is not guaranteed to be a real and positive scalar, consequently $\boldsymbol{f}_u$ is not optimized in terms of the BER performance. Computing $\boldsymbol{f}_u$ as in (Chae *et al.*, 2006) also impacts the convergence because the algorithm does not compare optimal beamformers to determine when the iterative process must stop.

III.4 Non-uniqueness of the Beamformers to Cancel Multiuser Interference

The beamformers that cancel the interference in the iterative process of CBF-SVD and maximize the SINR are not unique, quite the opposite there is a family of vectors $\boldsymbol{f}_u e^{-j\theta_u}$ $(0 \leq \theta_u \leq 2\pi)$ for each beamformer, as is shown next.

Lemma III.1 *If $\boldsymbol{f}_u$ is orthogonal to $\left\{\boldsymbol{w}_\ell^H \boldsymbol{H}_\ell\right\}_{\ell=1,\ell\neq u}^{K}$, then $\boldsymbol{f}_u e^{-j\theta_u}$ is also orthogonal to it for all θ_u.*

Proof. The proof of this lemma is straightforward by applying the properties of the dot product of vectors and the norm properties (Horn and Johnson, 1985) to the complex angle between two vectors. Consider $l \neq u$ and the product $\boldsymbol{w}_\ell^H \boldsymbol{H}_\ell \boldsymbol{f}_u$, then from the equation for the complex angle between two vectors (Scharnhorst, 2001) we have:

$$\frac{\left\langle \boldsymbol{w}_\ell^H \boldsymbol{H}_\ell, \boldsymbol{f}_u e^{-j\theta_u}\right\rangle}{\|\boldsymbol{w}_\ell^H \boldsymbol{H}_\ell\|\|\boldsymbol{f}_u e^{-j\theta_u}\|} = 0$$
$$\frac{e^{-j\theta_u}\left\langle \boldsymbol{w}_\ell^H \boldsymbol{H}_\ell, \boldsymbol{f}_u\right\rangle}{\|\boldsymbol{w}_\ell^H \boldsymbol{H}_\ell\|\|\boldsymbol{f}_u\||e^{-j\theta_u}|} = 0$$

since $|e^{-j\theta_u}| = 1$ and moving $e^{-j\theta_u}$ to the right side the expression is reduced to

$$\frac{\left\langle \boldsymbol{w}_\ell^H \boldsymbol{H}_\ell, \boldsymbol{f}_u\right\rangle}{\|\boldsymbol{w}_\ell^H \boldsymbol{H}_\ell\|\|\boldsymbol{f}_u\|} = \left(e^{-j\theta_u}\right)(0) = 0 \tag{23}$$

From 23, we can see that the orthogonality is not affected by substituting $\boldsymbol{f}_u$ by $\boldsymbol{f}_u e^{-j\theta_u}$, thus if $\boldsymbol{f}_u$ is orthogonal to $\left\{\left(\boldsymbol{w}_\ell^H \boldsymbol{H}_u\right)^T\right\}_{\ell=1,\ell\neq u}^{U}$, $\boldsymbol{f}_u e^{-j\theta_u}$ is also orthogonal to $\left\{\left(\boldsymbol{w}_\ell^H \boldsymbol{H}_u\right)^T\right\}_{\ell=1,\ell\neq u}^{U}$. □

Lemma III.2 *The SINR achieved by using $\boldsymbol{f}_u$ or $\boldsymbol{f}_u e^{-j\theta_u}$ is the same.*

Proof. Since $e^{-j\theta_u}$ is a complex number with unitary absolute value, we have

$$
\begin{aligned}
\mathsf{SINR}_u &= \frac{\left|\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u e^{-j\theta_u}\right|^2}{\sigma^2} \\
&= \frac{\left|\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u\right|^2 |e^{-j\theta_u}|^2}{\sigma^2} \\
&= \frac{\left|\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u\right|^2}{\sigma^2}. \square
\end{aligned}
$$

III.4.1 Proposed Improvements to the CBF-SVD Algorithm

Given that the product $\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u$ determines the BER performance, the objective of BER optimization is to choose the optimum beamformer $\boldsymbol{f}_u e^{-j\theta_u}$ that ensures that the product $\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u e^{-j\theta_u}$ be real and positive.

Problem Statement: Find a phase $\theta_u \in \{0, \cdots, 2\pi\}$ that minimize the absolute value of the complex angle between the vectors $\boldsymbol{w}_u^H \boldsymbol{H}_u$ and $\boldsymbol{f}_u e^{-j\theta_u}$

$$
\theta_u = \arg\min_{\theta} \left| cos^{-1} \left\{ \frac{\left\langle \boldsymbol{w}_\ell^H \boldsymbol{H}_\ell, \boldsymbol{f}_u e^{-j\theta_u} \right\rangle}{\|\boldsymbol{w}_\ell^H \boldsymbol{H}_\ell\| \|\boldsymbol{f}_u e^{-j\theta_u}\|} \right\} \right|. \tag{24}
$$

The solution to (24) is a kind of equalization and the angle θ_u is uniquely determined by

$$
\theta_u = \begin{cases} tan^{-1}\left(Im(g_u)/Re(g_u)\right), & \text{if } Re(g_u) > 0 \\ tan^{-1}\left(Im(g_u)/Re(g_u)\right) + \pi, & \text{if } Re(g_u) < 0 \\ 0, & \text{otherwise} \end{cases}, \tag{25}
$$

where $g_u = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{f}_u$.

By considering the optimum $\boldsymbol{f}_u e^{-j\theta_u}$ we propose to modify the CBF-SVD algorithm as follows.

At the Base Station

1. Initialize the combining vectors for each user and obtain the beamformers as is done in the steps 1-3 of the CBF-SVD algorithm described in subsection III.2.1.

2. Optimize the beamformers by setting $\boldsymbol{f}_u = \boldsymbol{f}_u e^{-j\theta_u}$, where the θ_u parameter is calculated from (24).

3. Execute steps 4-5 of the CBF-SVD algorithm described in subsection III.2.1 to determine if the algorithm stops or continues the iterative process.

4. Execute steps 6-7 of the CBF-SVD algorithm and optimize the final beamformers $\left\{\tilde{\boldsymbol{f}}_u\right\}_{u=1}^{U}$ by substituting each one as $\tilde{\boldsymbol{f}}_u e^{-j\theta_u}$, where θ_u is calculated by replacing $\boldsymbol{w}_u$ with $\boldsymbol{H}_u \boldsymbol{c}_i$ and $\boldsymbol{f}_u$ with $\tilde{\boldsymbol{f}}_u$ in (24).

5. Send the codebook indexes that correspond to the codes obtained in the quantization procedure to the mobile stations through the limited feedforward link.

At the Mobile Station

1. Calculate the combining vector as

$$\boldsymbol{w}_u = \frac{\boldsymbol{H}_u \boldsymbol{c}_u}{||\boldsymbol{H}_u \boldsymbol{c}_u||}. \tag{26}$$

2. Apply the combining vector to the received data in order to recover the transmitted data.

Our improved method achieves the same sum rate as CBF-SVD because the same SINR is achieved. The complexity increases in each iteration by the included optimization, however the necessary number of iterations is lower, and our proposal is optimized also in terms of BER performance.

III.4.2 Comments about computational complexity

The overall computational complexity of improved CBF-SVD algorithm is mainly determined by the number of the effective channel matrix singular value decompositions. By taking on account that it is necessary to perform U singular value decompositions of $\left\{\tilde{\boldsymbol{H}}_u\right\}_{u=1}^{U}$, whose computational complexity can be expressed as $U \cdot O(min((U-1)N_r^2, (U-1)^2 N_r))$ which is found in (Chae *et al.*, 2008).

III.5 Simulation Results

Monte Carlo simulation results are presented in this section to demonstrate the performance of the improved CBF-SVD algorithm. The first subsection demonstrates its performance improvements in terms of average BER, in i.i.d. complex Gaussian channel and with perfect channel knowledge. The second subsection shows the performance of the iterative process in terms of average necessary number of iterations to converge. we consider the assumptions of chapter II for all simulations in the next subsections.

III.5.1 Bit Error Rate Performance

Figure 9 shows a comparison of the BER performance of a multiuser system where the BS and each user have four antennas. We consider that the number of users is equal to the number of antennas at the BS and we use QPSK modulation to compute the BER performance. Six cases are studied:

(a) unquantized original CBF-SVD (obtained by executing steps 1-5 of the algorithm described in subsection III.2.1),

(b) unquantized improved CBF-SVD (calculated by executing steps 1-3),

(c) quantized original CBF-SVD,

(d) quantized improved CBF-SVD,

(e) greedy-based quantization (Chae *et al.*, 2008),

(f) Iteration-Based Independent Quantization (IBIQ) (Chae *et al.*, 2008).

It is observed that our proposal has better BER for both quantized and non-quantized versions of CBF-SVD. The phase error in the original CBF-SVD algorithm does not allow to achieve good BER performance, and correcting the resulting erroneous bits is almost impossible, even with error correction coding.

Both quantized CBF-SVD and improved quantized CBF-SVD show degradation in BER performance in comparison with its non-quantized versions, due to the quantization accuracy. As can be observed from figure 9, the BER achieved by quantized improved CBF-SVD amply outperforms greedy-based quantization, and it is very close to that of IBIC. A loss in BER is observed when using four-bit codes instead of six-bit codes, as expected.

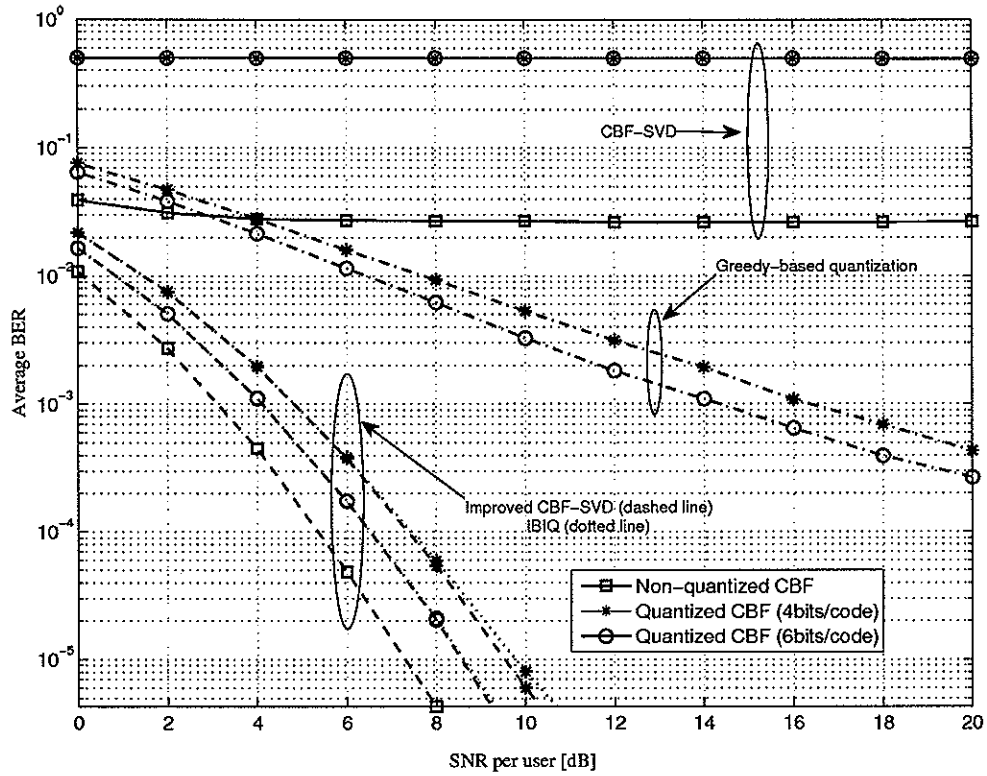

Figure 9. SNR vs BER for both non-quantized and quantized CBF.

Figure 10 shows the BER performance for the case of a six-bit codebook, for the cases when $N_t = N_t = U$ and $U = 2, 3, 4$. It can be seen that the average achieved BER decreases as the number of antennas and users increases, this is mainly due to the increase in the channel capacity. The worst performance is achieved by greedy-based quantization which shows no improvement as the number of antennas is increased, for SNR values greater than 16 dBs. As in figure 9, there is a marginal gap between improved CBF-SVD and IBIQ.

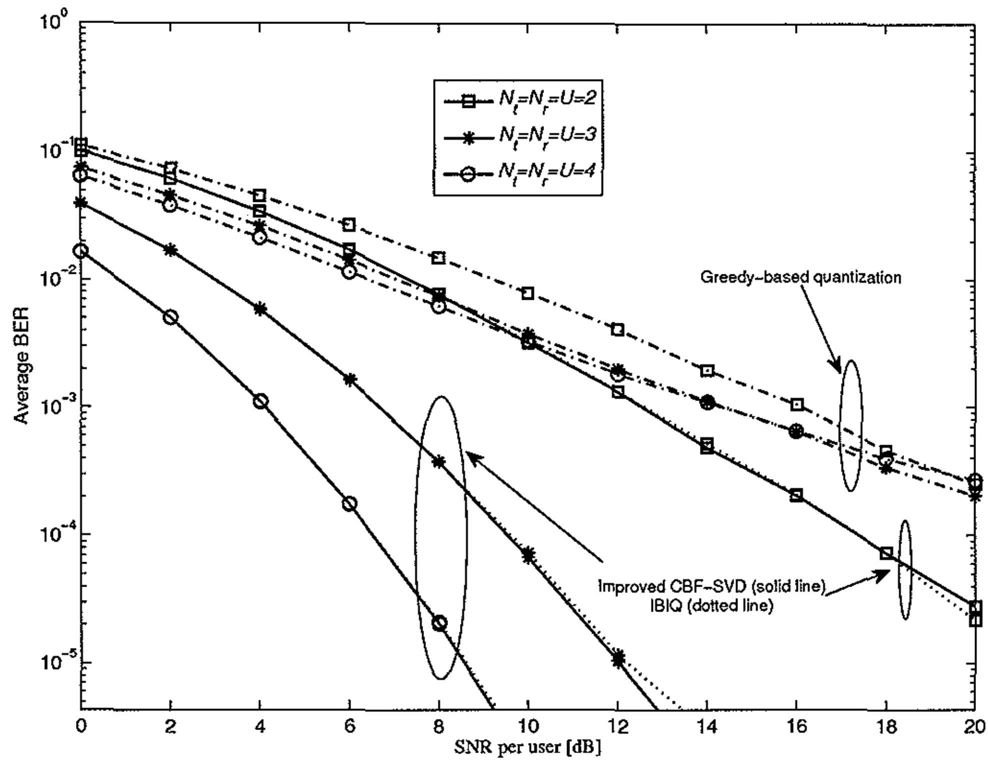

Figure 10. BER performance of improved CBF-SVD.

Figure 11 shows a comparison of the coded BER performance of both CBF-SVD and improved CBF-SVD, when $N_t = N_t = U = 4$. For channel coding, we used a convolutional code with generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$ with coding rate 1/2. The frame length was 30 bits (60 bits after convolutional encoding). Coded improved CBF-SVD outperforms coded CBF-SVD and exhibits a 3.5 dB gain over uncoded improved CBF-SVD. Coded CBF-SVD shows no significant improvement over uncoded CBF-SVD.

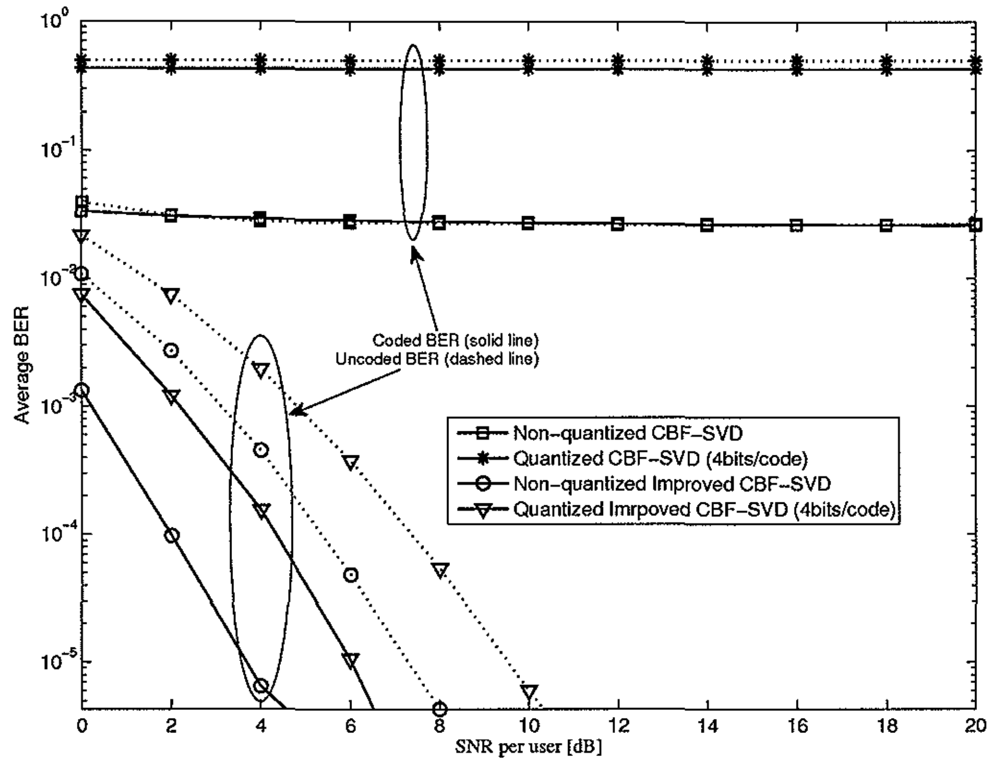

Figure 11. Coded BER performance of both CBF-SVD and improved CBF-SVD.

III.5.2 Convergence of the Algorithm

The convergence of both algorithms CBF-SVD and improved CBF-SVD is an important issue that can reduce the time of processing at the BS. This subsection provides numerical evaluation on the performance of improved CBF-SVD.

Figure 12 shows average iterations versus number of users, we consider $N_r = N_t = U$ where $U = 2, 3, 4, 5$. We examine the performance under different values for the stop criteria ϵ with 500 as the maximum number of possible iterations.

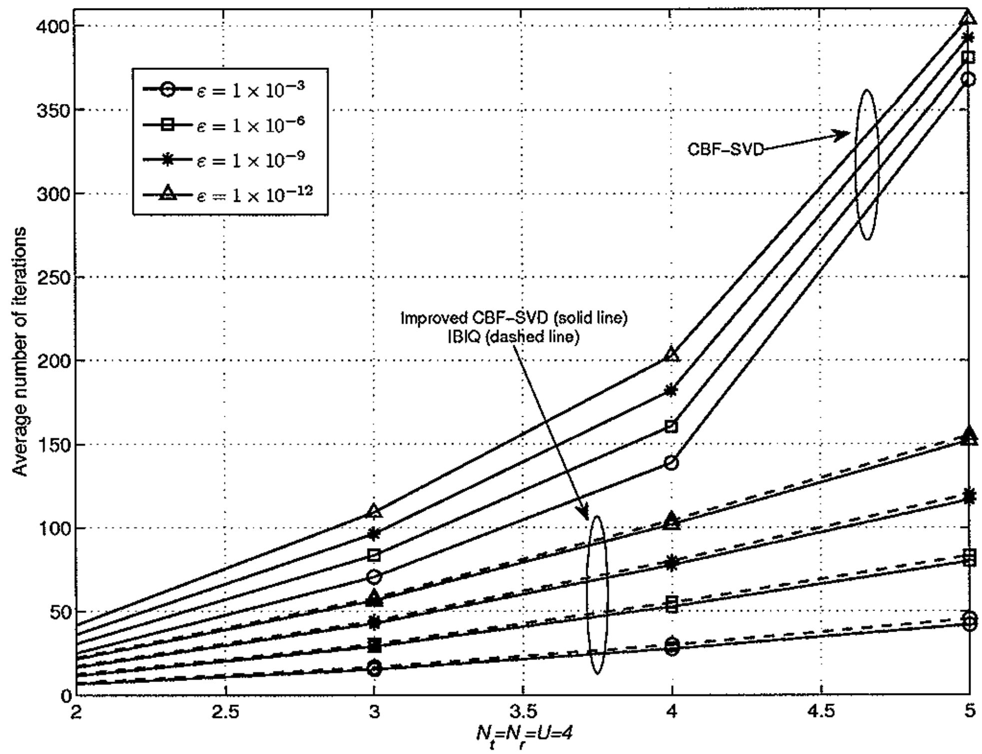

Figure 12. Convergence of both CBF-SVD and improved CBF-SVD algorithms.

In Figure 12, we can see from the curves that our proposal achieves the convergence faster than the original CBF-SVD for all values of ϵ. For the case $N_r = N_t = U = 5$ our method converges with an average reduction of 62% in the iterations, for the other cases it converges with almost 50% less than the average iterations computed by original CBF-SVD. Improved CBF-SVD slightly outperforms IBIQ for all considered cases.

III.5.3 Conclusions

In this chapter an improved version of CBF-SVD was presented. The proposed optimization to the CBF-SVD algorithm offers a better performance in terms of bit error rate with a lower number of iterations.

The improvement in the number of iterations is in the order of 50% for $N_r = N_t = U = 4$. We observed that for 10 dB of SNR in $N_r = N_t = U = 4$ configuration, improved CBF-SVD achieves an average BER of $6x10^{-6}$ (with a 6 bit codebook) while CBF-SVD achieves an average BER of 0.5 for all SNR values.

Chapter IV

Overhead Reduction in Single User MIMO-OFDM with Limited Feedback

IV.1 Introduction

In broadband (frequency selective) channels (figure 13), OFDM is used to facilitate equalization (Bölcskei, 2006). OFDM used together with MIMO (MIMO-OFDM), effectively divides the MIMO frequency selective channel into parallel flat-fading MIMO channels.

A MIMO-OFDM transmitter computes the Inverse Discrete Fourier Transform (IDFT) of the data previously converted from serial-to-parallel, and inserts a Cyclic Prefix (CP), with length equal or longer than the delay spread of the channel. At the receiver, the CP is eliminated and it is computed the Discrete Fourier Transform (DFT) of the resulting data (see figure 14). A good explanation on how OFDM converts a frequency selective channels in a set of flat fading channels can be found in (Paulraj *et al.*, 2003) and (Cho *et al.*, 2010).

In this chapter we provide a survey of algorithms for feedback/forward reduction that are based on the instantaneous channel state, our results were published in (Soriano-Equigua *et al.*, 2010b).

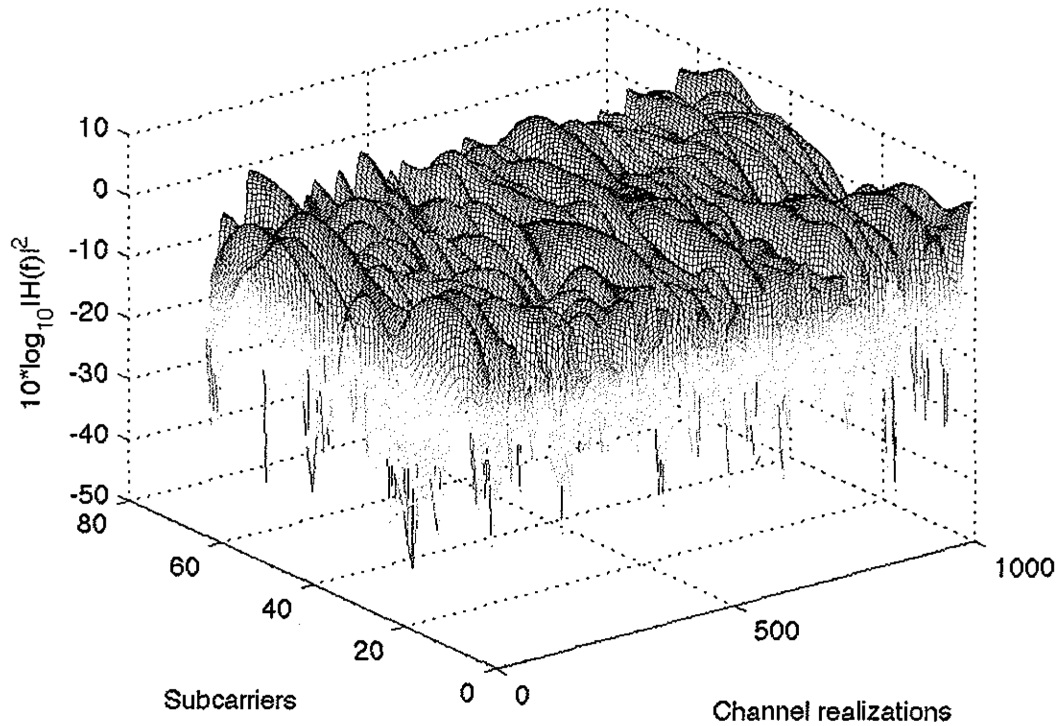

Figure 13. Frequency selective channel.

IV.2 System Model

Consider the downlink transmission of a single user MIMO-OFDM system with N subcarriers, N_t transmit antennas, and N_r receive antennas, as is illustrated in figure 14. A complex symbol $x[n]$ is transmitted at the nth ($1 \leq n \leq N$) subcarrier and it is multiplied by a transmit beamforming vector $\boldsymbol{f}[n]$. The codebook index that the MS will use to compute its combining vector is sent to the BS through a limited feedback link, which is assumed with zero delay. In this scheme all processing is done at the receiver, the BS just applies the beamformer that the MS indicate.

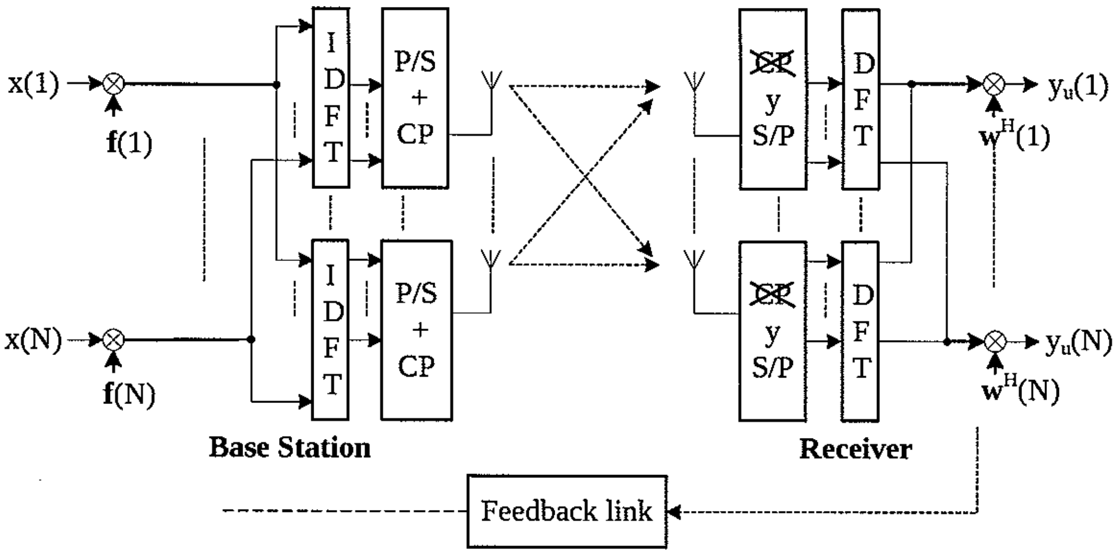

Figure 14. Single user limited feedback system model.

The signal $y[n]$ received by the MS at the nth subcarrier after removing the CP and processing with the combining vector $\boldsymbol{w}[n]$ is given by

$$y[n] = \boldsymbol{w}^H[n]\boldsymbol{H}[n]\boldsymbol{f}[n] + \boldsymbol{w}^H[n]\boldsymbol{v}[n], \tag{27}$$

where $\boldsymbol{v}[n]$ is a vector of independent and identically distributed (i.i.d.) complex zero-mean Gaussian noise with variance σ_v^2, $\boldsymbol{f}[n]$ and $\boldsymbol{w}[n]$ are unit norm vectors, and $\boldsymbol{H}[n]$ is the channel for the nth subcarrier represented by a matrix of size $N_r \times N_t$. The transmit beamforming vectors and receive combining vectors are calculated by computing the SVD of the channel matrix (Love *et al.*, 2003) (Mukkavilli *et al.*, 2003), which is summarized as follows, applied to each subcarrier n.

1. The receiver estimates its own channel matrix $\boldsymbol{H}[n]$ and performs the SVD of $\boldsymbol{H}[n]$.

$$[\boldsymbol{U}[n] \quad \boldsymbol{D}[n] \quad \boldsymbol{V}[n]] = svd(\boldsymbol{H}[n]) \tag{28}$$

2. The optimum transmit beamformer is the right singular vector corresponding to maximum singular value of $\boldsymbol{H}[n]$.

$$\boldsymbol{f} = \boldsymbol{V}[n]\,[:,1] \tag{29}$$

where $\boldsymbol{V}[n]\,[:,1]$ represents the first column vector of $\mathbf{V}[\mathrm{n}]$.

3. Obtain the quantized beamformer $\hat{\boldsymbol{f}}[n]$ by selecting the code that maximizes the channel effective gain.

$$\hat{\boldsymbol{f}}[n] = \arg\max_{c_i \in C} \quad \|\boldsymbol{H}[n]\boldsymbol{c}_i\|^2 \tag{30}$$

where C is a Grassmannian codebook (Love *et al.*, 2003).

4. The receiver send to the BS the codebook index corresponding to the quantized beamformer.

5. The BS applies the beamformer computed in equation (30) and the MS applies the combining vector given by

$$\boldsymbol{w}[n] = \frac{\boldsymbol{H}[n]\hat{\boldsymbol{f}}[n]}{\|\boldsymbol{H}[n]\hat{\boldsymbol{f}}[n]\|}, \tag{31}$$

IV.3 Techniques to Reduce Overhead in Limited Feedback

As it was illustrated in chapter I, both techniques limited feedforward and limited feedback are similar, having as main difference the direction of the information in the low rate link; techniques to reduce the overhead in limited feedback can be applied in limited feedforward. There are proposals to reduce the feedback overhead reported in the literature assuming different scenarios, for example: for temporally-correlated channels (Roh and Rao, 2004) (Banister and Zeidler, 2003), for spatially-correlated channels (Mondal and Heath, 2006), and for uncorrelated (spatially and temporary) channels (Zhou *et al.*, 2006) (Choi and Heath, 2005) (Mondal and Heath, 2005). In this work we consider the uncorrelated channels scenario.

To evaluate the performance of the main limited feedback proposals in MIMO-OFDM systems we analyze the BER performance for the single user case. We compare clustering (Mondal and Heath, 2005) (Choi and Heath, 2005), interpolated beamforming (Choi and Heath, 2005), recursive reduction feedback (Zhou *et al.*, 2006), and trellis reduction fedback (Zhou *et al.*, 2006). These algorithms are reviewed in the following subsections.

IV.3.1 Feedback Based on Clustering

Clustering is the simplest method to reduce the overhead in the feedback link. This proposal groups a number of subcarriers (typically two, four or eight subcarirriers/group) and choice the subcarrier located at the center of the group. As the number of subcarriers/cluster is even, we select the subcarrier that represent each cluster as $\{\hat{\boldsymbol{f}}[mK + \lceil K/2 \rceil]\}; \quad 0 \leq m \leq N/K - 1$, where m is the cluster index and K is the cluster size.

The beamformer corresponding to the selected subcarrier is set as beamformer for subcarriers in the cluster. Then, the combining vector is computed as

$$\boldsymbol{w}[n] = \frac{\boldsymbol{H}[n]\hat{\boldsymbol{f}}[mK + \lceil K/2 \rceil]}{\|\boldsymbol{H}[n]\hat{\boldsymbol{f}}[mK + \lceil K/2 \rceil]\|}, \tag{32}$$

where $\hat{\boldsymbol{f}}[mK + \lceil K/2 \rceil]$ is beamformer for the central subcarrier and k is a subcarrier in the cluster.

Assuming that C is the codebook to quantize the transmit beamformers and considering a fixed cluster size, the number of necessary feedback bits is $(N/K)b$, where b is the number of bits/code.

IV.3.2 Feedback Based on Interpolation

In (Choi and Heath, 2005) was proposed a modified spherical linear interpolator to calculate the beamformers in the subcarriers group. This interpolator incorporates a phase parameter θ that is optimized in terms of BER. The interpolator is given by

$$\bar{f}\Big[(mK+k),\theta[m]\Big] = \frac{(1-p_k)\,f[mK+1] + p_k e^{j\theta[m]} f[(m+1)K+1]}{\|(1-p_k)\,f[mK+1] + p_k e^{j\theta[m]} f[(m+1)K+1]\|}, \tag{33}$$

where $p_k = (k-1)/K$ is the interpolation weight factor, k $(1 \leq k \leq K)$ is the subcarrier in the mth cluster, and $\theta[m]$ is the phase parameter for the mth cluster. In this chapter we consider the $\theta[m]$ optimization as

$$\theta[m] = \arg\min_{\theta\in\Theta} \quad \|H[mK+n]\bar{f}[mK+n]\|, \tag{34}$$

where $\Theta = \left\{0, \frac{2\pi}{P}, \cdots, \frac{2(P-1)\pi}{P}\right\}$, and P is the number of quantized levels.

The necessary feedback bits of interpolated beamforming is $(N/K)b + (N/K)b_{\Theta}$, where b_{Θ} is the number of bits needed to quantize Θ.

IV.3.3 Recursive Feedback Reduction

This proposal is based on recursive vector quantization (Gersho and Gray, 1992). It uses state variables to summarize the influence of the past quantization on the current operation of the quantizer. This algorithm can be summarized as follows:

1. the subcarrier number is used as time unity,

2. each subcarrier can have 2^b possible states, where b is the number of bits/code,

3. it is assumed as initial state for the first subcarrier the result of applying equation (30),

4. for the second subcarrier, it is selected as possible next 2^{b_2} states those codes with less chordal distance from the code obtained in step 3, where b_2 are the bits to denote the possible neighbors. The quantized beamformer for this subcarrier is computed as the next state that maximizes equation (30),

5. the process is the same for the next subcarriers.

The feedback overhead of this method is $b + (N - 1)b_2$ bits.

IV.3.4 Trellis Feedback Reduction

This method is a generalization of recursive feedback reduction. Trellis feedback reduction is summarized as follows

1. A trellis is built assuming 2^b possible states for each subcarrier,

2. the state corresponding to the first subcarrier is initialized as in subsection IV.3.3,

3. as in recursive feedback reduction, the next 2^{b_2} states are computed,

4. it is computed for each next state its own next states and so on, until the full trellis is completed. The metric to pass from one state to other state is given by the theoric BER of the used modulation, we use QPSK for our simulations.

5. The optimum codebook indexes are chosen from the trellis path that minimizes the BER performance.

The feedback overhead of this method is also $b + (N - 1)b2$ bits.

IV.4 Simulation Results

In all simulations it is assumed that

(i) $N = 64$, $N_t = 4$, $N_r = 4$,

(ii) the digital modulation is QPSK,

(iii) the channel between different pair of antennas are independent (the channel B of Hiperlan2 (Medbo and Schramm, 1998) was considered),

(iv) the receiver estimates perfectly its own channel,

(v) both BS and MS share the same codebook,

(vi) the channel is assumed constant during a frame transmition.

IV.4.1 Performance for a Fixed Codebook Size

In this subsection the reviewed methods are compared subject to use the same number of bits to quantize the beamforming vectors. A codebook size of six bits is considered; for the case of interpolated beamforming two additional bits are considered (for the phase parameter). For clustering and interpolated beamforming eight subcarriers/cluster is assumed. For recursive feedback reduction and Trellis feedback reduction four neighbors were considered.

It is observed from figure 15 that the unquantized beamforming performs better than the other algorithms. A loss due to quantization is observed, the best quantized algorithm in figure 15 is the interpolated beamforming at the cost of two additional bits for its phase parameter. The method with worst performance is recursive feedback reduction.

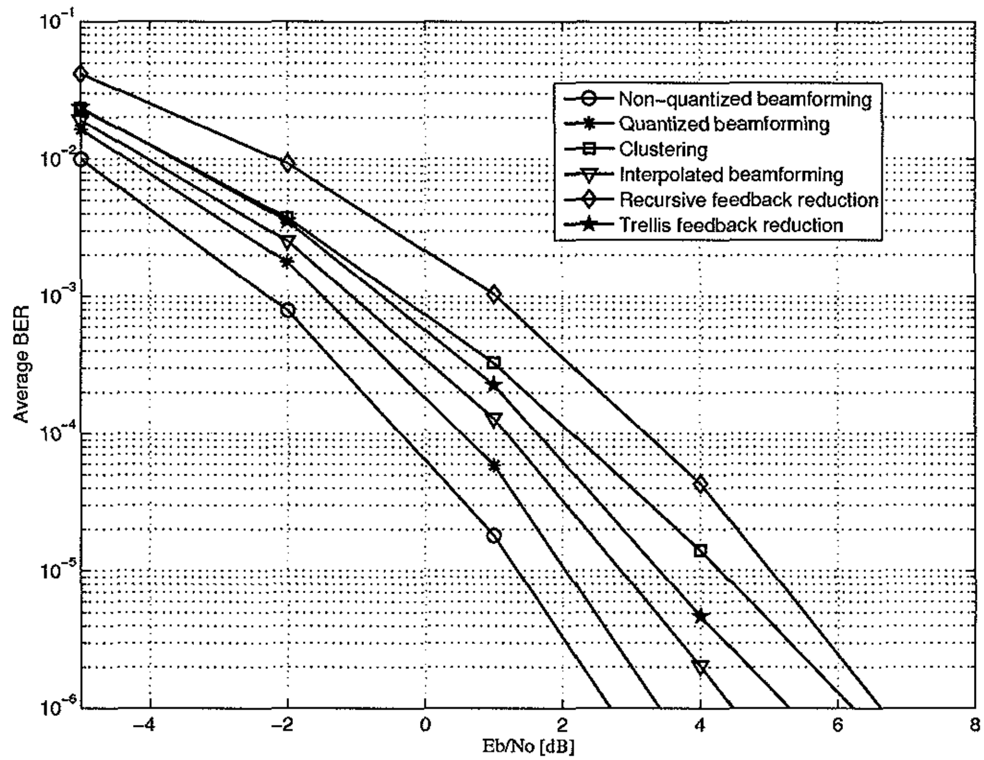

Figure 15. BER versus Eb/No in single user MIMO with limited feedback.

IV.4.2 Performance for Fixed Overhead

For a fair comparison, we analyze the performance for a fixed number of feedback bits. Table I shows the feedback settings, both recursive feedback reduction and trellis feedback reduction use 65 bits.

Table I. Feedback settings

Method	Group Size	Number of Groups	Codebook Size
Quantized beamforming	1	64	2
Clustering	2	32	4
	4	16	16
	8	8	256
Interpolated beamforming	4	16	4
	8	8	64
Recursive feedback reduction	64	1	4
Trellis feedback reduction	64	1	4

From figure 16 it is observed that the interpolated beamfoming (with six bits/code, 2-bits phase parameter, and eight subcarrriers/group) achieves the best performance. As the Eb/No increases, the clustering method (with four bits/code) approaches the interpolated beamforming with a minor complexity. On the other hand, the methods with worst performance are recursive feedback reduction and trellis feedback reduction, these methods suffer major loss because they use two bits/code.

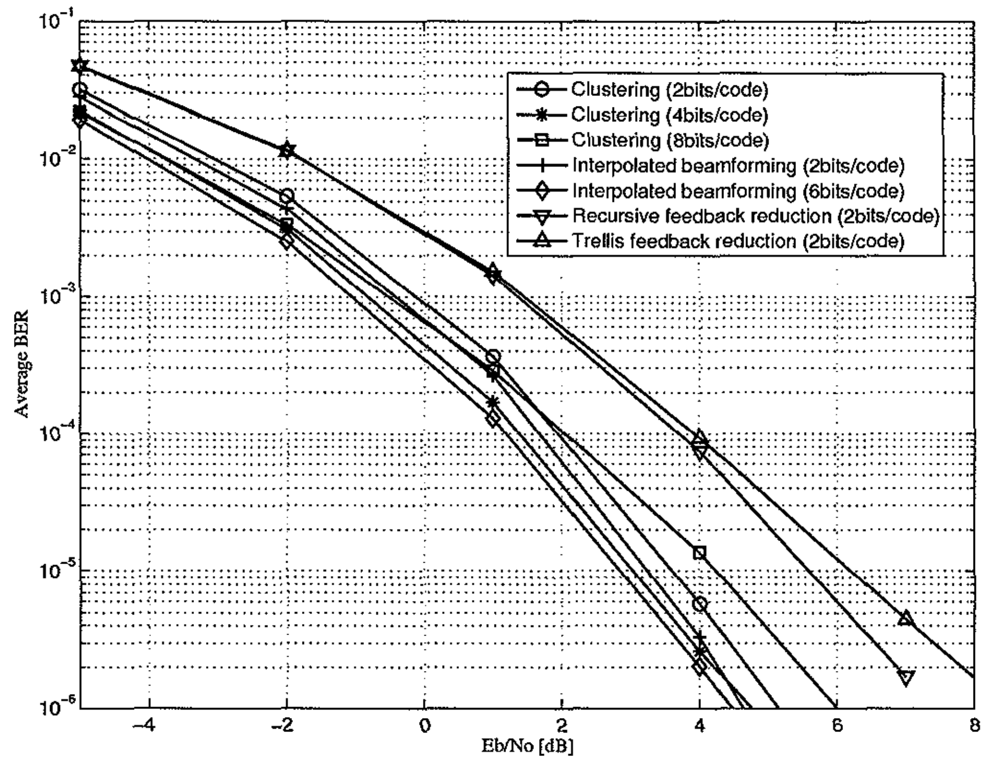

Figure 16. Fair comparison of limited feedback algorithms.

IV.5 Conclusions

This chapter presents a comparison in terms of BER performance and overhead reduction of the main proposals in the literature to reduce the feedback requirements in a single user closed loop MIMO-OFDM system.

It was demonstrated that the best algorithm with low feedback overhead was interpolated beamforming. Clustering obtains a performance near that of interpolated beamforming, with less complexity.

Chapter V

Coordinated Beamforming for MU-MIMO-OFDM

V.1 Introduction

CBF can be applied in multiuser MIMO-OFDM systems by applying the narrow-band design in (Chae *et al.*, 2008) for each subcarrier. CBF can also be implemented using the proposals described in chapters II and III. As in single user MIMO-OFDM with limited feedback case, the total overhead increases linearly with the number of subcarriers. As a result, straightforward application of the CBF algorithm can cause excess feedforward overhead in MIMO-OFDM systems.

Subcarrier grouping techniques have been previously considered for LTE as in (Texas Instruments, 2006b), where they deal with chunks of 25 subcarriers and chose one single channel matrix for representing up to 4 chunks. Grouping has also been considered in (Texas Instruments, 2006a), where they mention that the sum rate, or the maximum throughput among the subcarrier blocks, is considered for choosing the best code. However, most work in MU-MIMO in 3GPP LTE has been aimed to FDD (Duplicy *et al.*, 2011).

In this chapter we present four strategies to implement CBF in MU-MIMO-OFDM systems. We provide an explanation of the system model and a review of the CBF algorithm used as a basis of the development of our proposals. We present the analysis of simulation results divided in two parts in order to emphasize our published results in (Soriano-Equigua *et al.*, 2011b).

V.2 System Model and CBF Review

Consider the downlink transmission of a multiuser MIMO-OFDM system with N sub-carriers, N_t transmit antennas, N_r receive antennas, and U users as illustrated in Fig. 17.

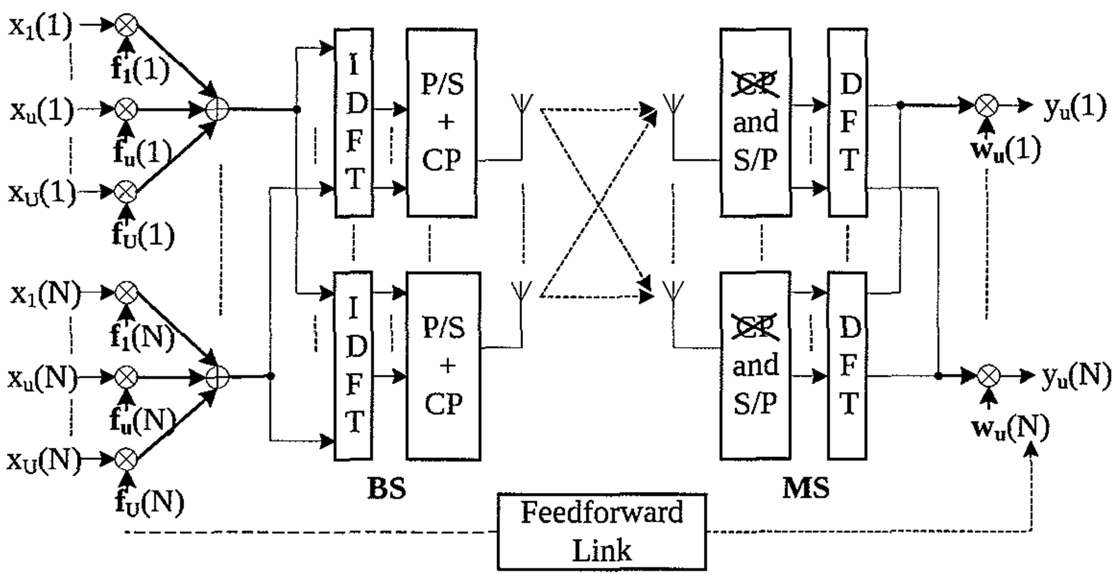

Figure 17. CBF for MIMO-OFDM system model with limited feedforward.

A complex symbol $x_u[n]$ transmitted by the uth $(1 \leq u \leq U)$ user at the nth $(1 \leq n \leq N)$ subcarrier is multiplied by a transmit beamforming vector $\boldsymbol{f}_u[n]$, then added to the beamformed data that belong to remaining users. The necessary information to calculate the combiners is sent to each user through a limited feedforward link, which is assumed with zero delay.

We assume that the length of the cyclic prefix is longer than the maximum path delay of the frequency selective channel between the BS and the MSs. The channel is assumed to be constant during the OFDM symbol transmission time.

The signal $y_u[n]$ received by the uth user at the nth subcarrier after removing the CP and processing with the combining vector $\boldsymbol{w}_u[n]$ is given by

$$y_u[n] = \boldsymbol{w}_u^H[n]\left(\boldsymbol{H}_u[n]\sum_{\ell=1}^{U}\boldsymbol{f}_\ell[n]x_\ell[n] + \boldsymbol{v}_u[n]\right), \tag{35}$$

where $\boldsymbol{v}_u[n]$ is a vector of independent identically distributed (i.i.d.) complex zero-mean Gaussian noise with variance σ_v^2, $\boldsymbol{f}_u[n]$ and $\boldsymbol{w}_u[n]$ are unit norm vectors, and $\boldsymbol{H}_u[n]$ is the channel for the nth subcarrier at the uth MS represented by a matrix of size $N_r \times N_t$. We assume that the transmit power is equally allocated across all users.

The transmit beamforming vectors and receive combining vectors are calculated by using the matrix inversion approach for CBF described in (Chae *et al.*, 2008), which is summarized as follows, applied to each subcarrier n.

1. Initialize each receive combining vector by setting it equal to the left singular vector of $\boldsymbol{H}_u[n]$ that corresponds to the maximum singular value. Other initializations are also possible.

2. Form the effective channel matrix as

$$\boldsymbol{H}_{\text{eff}}[n] = \left[\left(\boldsymbol{w}_1^H[n]\boldsymbol{H}_1[n]\right)^T \cdots \left(\boldsymbol{w}_U^H[n]\boldsymbol{H}_U[n]\right)^T\right]^T. \tag{36}$$

3. Calculate the beamforming vectors $\boldsymbol{f}_u[n]$ such that they cancel the interference between users, by taking the normalized columns of the pseudo inverse of $\boldsymbol{H}_{\text{eff}}[n]$ as follows

$$\left[\boldsymbol{f}_1[n] \cdots \boldsymbol{f}_u[n] \cdots \boldsymbol{f}_U[n]\right] = \text{cols}\left(\frac{\boldsymbol{H}_{\text{eff}}^{\dagger}[n]}{\|\boldsymbol{H}_{\text{eff}}^{\dagger}[n]\|}\right). \tag{37}$$

4. Check the stopping criteria. The algorithm stops when the difference between the previous and current beamformers is small enough. Otherwise update the combining vectors using MRC

$$\boldsymbol{w}_u[n] = \boldsymbol{H}_u[n]\boldsymbol{f}_u[n], \tag{38}$$

and go to step 2.

5. Quantize the beamformers. The beamformers are complex vectors that can take an infinite number of values. To send them through a limited bandwidth channel, they need to be quantized. We use a finite set of beamforming vectors from what is called a codebook; actually, Grassmannian codebooks (Love *et al.*, 2003) were used on the simulations. To find the quantized beamformers $\hat{\boldsymbol{f}}_u[n]$, the transmitter selects the codebook index corresponding to the code that maximizes the signal-to-interference-plus-noise ratio (SINR)

$$\hat{\boldsymbol{f}}_u[n] = \arg\max_{\boldsymbol{c}_i \in C} \quad \text{SINR}_u[n, \boldsymbol{c}_i], \tag{39}$$

where

$$\text{SINR}_u[n, \boldsymbol{c}_i] = \frac{|\boldsymbol{c}_i^H \boldsymbol{R}_u[n] \boldsymbol{f}_u[n]|^2}{\boldsymbol{c}_i^H \left(\sum_{l=1, l\neq u}^{U} \boldsymbol{R}_u[n] \boldsymbol{f}_l[n] \boldsymbol{f}_l^H[n] \boldsymbol{R}_u[n] \right) \boldsymbol{c}_i + \boldsymbol{c}_i^H \boldsymbol{R}_u[n] \boldsymbol{c}_i \sigma^2}, \tag{40}$$

$\boldsymbol{R}_u[n] = \boldsymbol{H}_u^H[n]\boldsymbol{H}_u[n]$ is the matched channel matrix and C is the codebook.

6. Cancel residual interference. After quantization, the BS updates the combiners, executes steps two and three again to mitigate residual interference due to the quantization operation.

7. Send to the MSs their codebook index calculated in (39). The codebook index corresponds to the user's quantized beamforming vector, necessary to compute the combining vector by MRC.

V.3 CBF for MIMO-OFDM with Limited Feedforward

In this section, we extend the clustering algorithm ((Choi and Heath, 2005), (Mondal and Heath, 2005)) and interpolated beamforming of (Choi and Heath, 2005) to CBF for MIMO-OFDM. Also, we propose adaptive clustering and advanced clustering methods to reduce the feedforward overhead and maximize the sum rate for the CBF in MIMO-OFDM systems.

V.3.1 Clustering for Coordinated Beamforming

It is well known that in MIMO-OFDM systems, the channels are correlated (Liu and Jafarkhani, 2007); as the beamforming vectors depend on the channel, they are also correlated. Figure 18 shows the beamformers correlation under different Root Mean Square (RMS) delay spread values of the wireless channel, the beamformers were calculated using the steps 1-4 of the algorithm described in Section V.2. From the figure 18 we observed that there is a higher correlation for a RMS delay spread value of $50ns$, which usually corresponds to indoor channels.

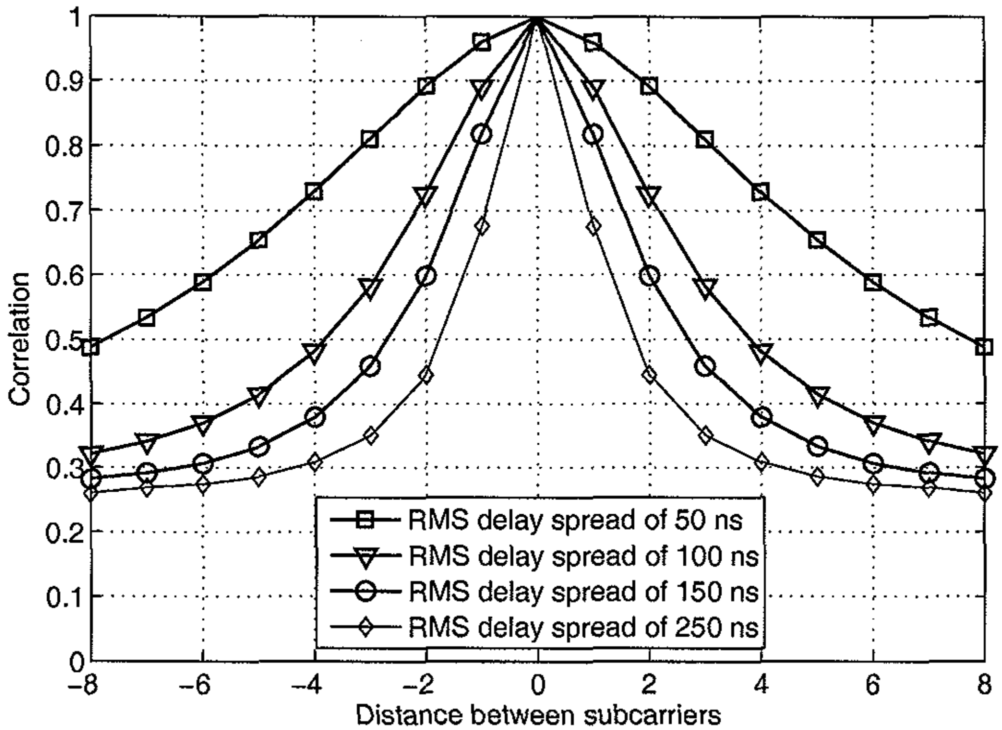

Figure 18. Correlation of the unquantized beamforming vectors.

To exploit the beamformers correlation, we propose a clustering technique where K adjacent subcarriers are grouped and a single beamformer is selected to represent all subcarriers of the cluster. We use the beamforming vector corresponding to the center subcarrier in the cluster. A summary of the algorithm is given below.

At the Base Station

1. Compute the quantized beamforming vectors corresponding to the selected subcarri-

ers $\{\hat{\boldsymbol{f}}_u[mK + \lceil K/2 \rceil]\}$; $\quad 0 \le m \le N/K - 1$, using the CBF algorithm described in Section V.2; where m is the cluster index.

2. Calculate the combining vectors of all subcarriers in the group applying

$$\boldsymbol{w}_u[mK + k] = \boldsymbol{H}_u[mK + k]\hat{\boldsymbol{f}}_u[mK + \lceil K/2 \rceil] \tag{41}$$

where k is the subcarrier index ($1 \le k \le K$) in the mth cluster.

3. Mitigate residual inter-user interference due to clustering and quantization using steps two and three described in section V.2.

4. Feedforward the codebook indices, corresponding to the quantized beamforming vectors, to the receivers.

At the Mobile Station

1. Find the combining vectors using

$$\boldsymbol{w}_u[mK + k] = \frac{\boldsymbol{H}_u[mK + k]\tilde{\boldsymbol{c}}_u[m]}{\|\boldsymbol{H}_u[mK + k]\tilde{\boldsymbol{c}}_u[m]\|}, \tag{42}$$

where k is the subcarrier index ($1 \le k \le K$) in the cluster m.

2. Apply the combining vector to the received data in each subcarrier.

As a result of the clustering algorithm we have a N/K overhead reduction per user in the feedforward link, subject to impose zero interference between users.

V.3.2 Interpolation Based Coordinated Beamforming

The correlation exhibited in the beamforming vectors can be exploited to implement beamformer interpolation. A modified spherical linear interpolation has been proposed for the single user MIMO-OFDM beamforming case (Choi and Heath, 2005) showing a good tradeoff between bit error rate performance and number of feedback bits. Interpolation was proposed using a phase parameter θ to improve performance under different criteria. With K as the size of a subcarrier group, the interpolated beamforming vector for the subcarrier number $(mK + k)$ (assuming module N addition) is given by (Choi and Heath, 2005):

$$\bar{\boldsymbol{f}}_u\Big[(mK+k), \theta_u[m]\Big] = \frac{(1-p_k)\boldsymbol{f}_u[mK+1] + p_k e^{j\theta_u[m]}\boldsymbol{f}_u[(m+1)K+1]}{\|(1-p_k)\boldsymbol{f}_u[mK+1] + p_k e^{j\theta_u[m]}\boldsymbol{f}_u[(m+1)K+1]\|}, \tag{43}$$

where $p_k = (k-1)/K$ is the interpolation weight factor.

Unlike in (Choi and Heath, 2005), where $\theta_u[m]$ parameter is optimized for single user case, we consider the impact of the inter-user interference such that $\theta_u[m]$ is jointly optimized by maximizing SINR in the subcarrier group as

$$\theta_u[m] = \arg\max_{\theta_u \in \Theta} \sum_{u=1}^{U} \sum_{n=mK+1}^{(m+1)K} \log_2\left(1 + \mathsf{SINR}_u\left[n, \theta_u\right]\right), \tag{44}$$

where $\Theta = \Big\{0, \frac{2\pi}{P}, \cdots, \frac{2(P-1)\pi}{P}\Big\}$, P is the number of quantized levels, and $\mathsf{SINR}_u[n, \theta_u]$ is found by substituting $\boldsymbol{c}_i$ in (39) by $\bar{\boldsymbol{f}}_u(mK + k; \theta_u[m])$.

The interpolated CBF algorithm is summarized as follows.

At the Base Station

1. Compute the quantized beamforming vectors corresponding to the selected subcarriers $\{\boldsymbol{f}_u[mK + 1]\}$, using CBF algorithm in Section II.2.

2. Interpolate the beamforming vectors for the subcarriers in the group, using (43) and (44).

3. Calculate the combining vectors of all subcarriers in the group using, interpolated beamforming vectors as

$$w_u[mK + k] = \boldsymbol{H}_u[mK + k]\bar{\boldsymbol{f}}_u[(mK + k), \theta_u[m]] \tag{45}$$

4. Mitigate residual inter-user interference due to interpolation and quantization using steps two and three described in section V.2.

5. Feedforward the codebook indices corresponding to the quantized beamforming vectors to the receivers.

At the Mobile Station

1. Using the first and last quantized beamforming vectors for each group, find the remaining beamforming vectors for all the subcarriers in the group, applying (43).

2. Apply (45) to find the combining vectors.

3. Apply the combining vector to the received data in each subcarrier.

Regarding algorithm complexity, it is important to mention that the phase optimization (for the interpolation algorithm) requires an extensive search over all codebook elements and over all the elements of Θ. Furthermore, the use of interpolation requires an extra overhead in feedforward bits for sending the phase information ($\lceil N/K \rceil b_\Theta$ bits per user).

V.3.3 Adaptive Clustering

The cluster size depends on the delay spread of the channel, which affects the correlation among adjacent subcarriers. This way the optimum cluster size depends on the actual channel realization. To determine the appropriate cluster size we propose the following adaptive method. Before transmitting each frame, the sum rate is calculated for each subcarrier group size, adjusting the corresponding codebook size for having a total feedforward bits. The group size that gives the maximum sum rate is chosen and informed to the users via a signaling message. The users know in advance which codebook to use for each subcarrier group size. If the maximum sum rate is obtained with the same group size used for the previous frame, no message is sent to the users. In order to save signaling overhead bits, we propose to send only the group number (in a broadcast message to all users), instead of the group size. So for the case of three group size, the signaling message may be as simple as sending 1, 2 or 3 using just two bits. The same adaptive method may be used for the interpolation case, with a further increase in the number of computations.

V.3.4 Advanced Clustering

In this subsection we propose an advanced clustering to exploit better the correlation between beamformers. Our proposal reduces the feedforward overhead as clustering algorithm in a MU-MIMO-OFDM system, but our algorithm achieves a higher sum rate. Also, Advanced clustering outperforms adaptive clustering and interpolated CBF. The results of this algorithm were presented in (Soriano-Equigua *et al.*, 2011b).

In advanced clustering, the transmitter quantizes the beamforming vector that maximizes the sum rate in the cluster and sends the index of the quantized vectors to each user. A summary of the algorithm is given below.

At the Base Station

1. Compute the quantized beamforming vectors for each subcarrier in each cluster by using the steps 1-5 of the CBF algorithm described in Section V.2. It is important to mention that for both clustering and advanced clustering, it is possible to apply others CBF algorithms, as those described in chapters II and III.

2. For the mth cluster ($0 \leq m \leq N/K-1$), construct U subcodebooks $\{\hat{C}_u[m]\}_{u=1}^{U} \subset C$, where the subcodebook $\hat{C}_u[m]$, that corresponds to the uth user, contains the K quantized beamformers of the subcarriers group.

3. Eliminate duplicated codes in each subcodebook such that $\hat{\boldsymbol{c}}_{u,i} \neq \hat{\boldsymbol{c}}_{u,j}, \forall i \neq j$ and $\hat{\boldsymbol{c}}_{u,i}, \hat{\boldsymbol{c}}_{u,j} \in \hat{C}_u[m]$.

4. Construct the effective channel matrix for each combination of codes

$$\hat{\boldsymbol{H}}_{\text{eff}}[n] = \left[\left(\boldsymbol{c}_{1,i_1}^H \boldsymbol{R}_1[n]\right)^T \cdots \left(\boldsymbol{c}_{U,i_U}^H \boldsymbol{R}_U[n]\right)^T\right]^T. \tag{46}$$

5. Compute the beamformers

$$\left[\tilde{\boldsymbol{f}}_1[n] \cdots \tilde{\boldsymbol{f}}_u[n] \cdots \tilde{\boldsymbol{f}}_U[n]\right] = \text{cols}\left(\frac{\hat{\boldsymbol{H}}_{\text{eff}}^{\dagger}[n]}{\|\hat{\boldsymbol{H}}_{\text{eff}}^{\dagger}[n]\|}\right). \tag{47}$$

6. Select the codes that maximize the sum rate in the cluster. The beamformers obtained in the previous step cancel the interference by using the codes $\boldsymbol{c}_{1,i_1}, \cdots, \boldsymbol{c}_{U,i_U}$ in (47) for the combining vectors calculation, then the sum rate equation doesn't have any term of

interference and the maximization is done as follows

$$\left[\tilde{c}_1[m]\cdots\tilde{c}_u[m]\cdots\tilde{c}_U[m]\right] = \underset{c_{1,i_1}\in\hat{C}_1[m]\cdots c_{U,i_U}\in\hat{C}_U[m].}{\arg\max} \sum_{u=1}^{U}\sum_{n=mK+1}^{(m+1)K}\log_2\left(1+\frac{|\boldsymbol{c}_{u,i_u}^H\boldsymbol{R}_u[n]\tilde{\boldsymbol{f}}_u[n]|^2}{\sigma^2}\right). \tag{48}$$

7. Obtain the codebook indexes from C and send them to the receivers. The final beamformers are the ones corresponding to the selected codes, computed in step 5.

At the Mobile Station

1. Calculate the combining vectors of all subcarriers in the group by applying equation (42).
2. Apply the combining vector to the received data in each subcarrier to recover the transmitted data.

V.3.5 Feedforward Reduction and Complexity

As a result of the advanced clustering algorithm we have a N/K overhead reduction per user in the feedforward link, subject to impose zero interference between users. The overhead reduction is similar to clustering, where the central subcarrier is selected to represent the cluster, however our proposal maximizes the sum rate in the cluster.

The complexity remains in the BS, where the optimization is done. The receivers just compute their combining vectors by using the codebook indexes. The BS executes N times the steps 1-5 of the algorithm described in Section II.2, the procedure to select the optimum codes under maximum sum rate criteria has variable complexity because it depends on the subcodebooks cardinality. The cardinality of each subcodebook is a function of the beamformers correlation, as the correlation increases we have more duplicated codes in the subcodebook and we obtain reduced subcodebooks; the worst case is given when there is no correlation between beamformers, and it is necessary to compare 2^{UK} sum rates per cluster.

V.4 Simulation Results

This section is divided in two subsections, in the first subsection we analyze the performance of the clustering, interpolated CBF, and adaptive clustering algorithms, in the second one we compare the advanced clustering with the proposed clustering algorithms. In both subsections we carried out a series of simulations, where we assume the following:

(i) The number of subcarriers $N = 64$,

(ii) the channels between different transmit and receive antenna pairs are independent,

(iii) the BS obtains a perfect channel estimate,

(iv) each MS correctly estimates its own channel

(v) all users share the same codebook.

To simulate the different channels, we used the channel profiles of hiperlan2 documented in (Medbo and Schramm, 1998) and (Ibnkahla, 2004) and the fading was generated according to (Zheng and Xiao, 2003).

V.4.1 Performance of Clustering, Interpolated CBF, and Adaptive Clustering

V.4.1.1 Quantized Coordinated Beamforming for Fixed Overhead Bits and System Parameters

Figure 19 shows sum rate results versus SINR for fixed system dimensions ($N_t = N_r = U = 4$) and fixed number of feedforward bits per user, using the Hiperlan2 B channel model (Medbo and Schramm, 1998). For a fair comparison, we adjusted the codebook size and the group size for both algorithms (see Table II). For the case of interpolated CBF considered 2 bits per subcarrier group for phase information. For comparison we plot the quantized CBF with $K = 1$ (no subcarrier grouping) and 4-bit codebook.

Table II. Feedforward settings

Method	Group Size	Number of Groups	Codebook Size
Quantized CBF	1	64	2
Clustering	2	32	4
	4	16	16
	8	8	256
Interpolated CBF	4	16	4
	8	8	64

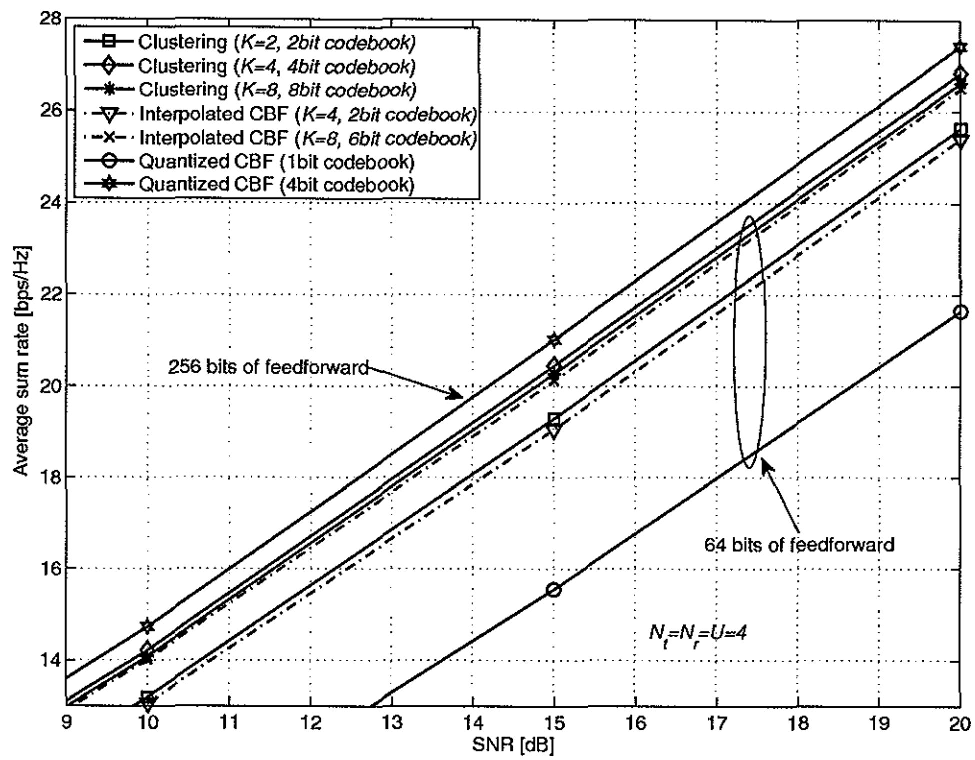

Figure 19. Sum rate versus SNR in CBF for MU-MIMO-OFDM.

We observe from the curves of 64 feedforward bits, that the best performance is achieved with advanced clustering CBF with $K = 4$ and 4 bit codebook, and the worst performance is observed for the case of no subcarrier grouping ($K = 1$) and 1 bit codebook. The maximum achieved sum rate by employing advanced clustering is 19.3% higher than the achieved sum rate by employing no subcarrier grouping with 1 bit codebook. We also observe that advanced clustering CBF gives a 75% savings on feedforward bits compared with quantized CBF (with 4 bit codebook), at a cost of losing 2.15% in sum rate.

V.4.1.2 Quantized Coordinated Beamforming for Fixed Overhead Bits and Different Channel Profiles

Figure 20 shows the sum rate versus the root-mean-square (RMS) delay spread for fixed dimensions ($N_t = N_r = U = 4$) and fixed number of feedforward bits per user. We consider the HiperLan/2 channel models A, B, C, and E (Medbo and Schramm, 1998) to show the performance of the proposed algorithms under different channel conditions. It is observed that adaptive clustering has better sum rate performance for all the channel profiles; cluster size 8 gives better performance with a RMS delay spread of 50 ns, because the correlation between subcarriers is high and decreases as the delay spread increases. Consequently, the optimum cluster size decreases as the delay spread increases.

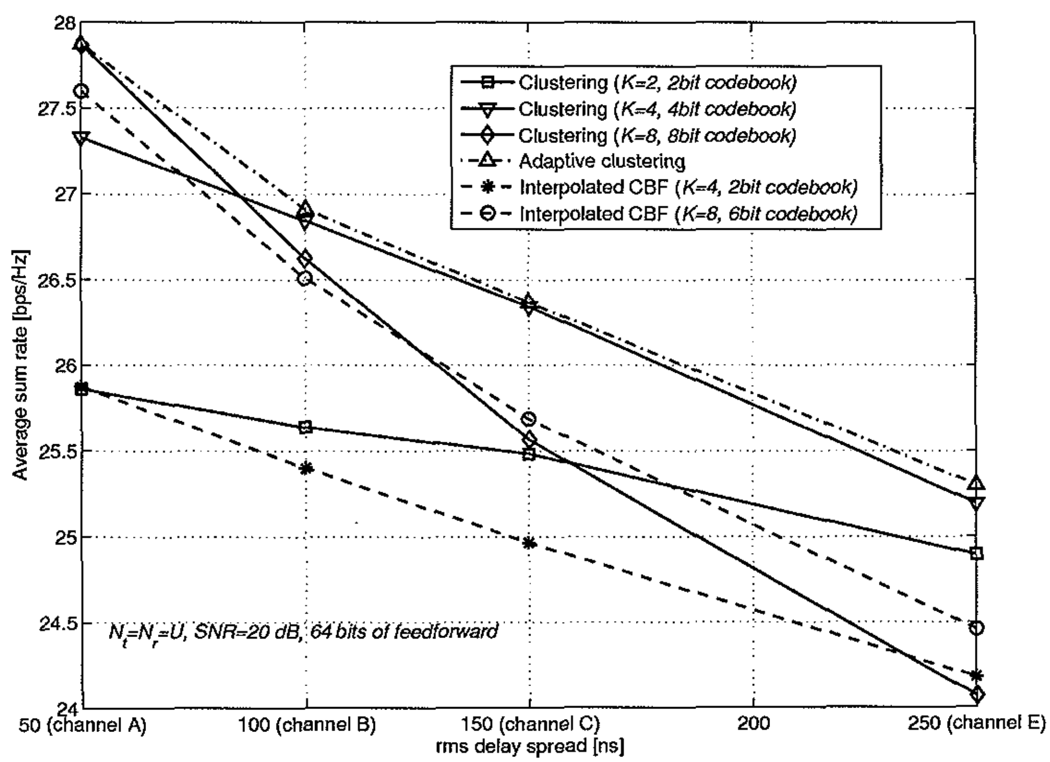

Figure 20. Sum rates as a function of the channel profile.

V.4.2 Performance of advanced clustering

V.4.2.1 Quantized CBF for Different System Parameters

Figure 21 shows sum rate versus SINR results for fixed system dimensions ($N_t = N_r = U = 4$) and fixed number of feedforward bits per user, using the Hiperlan2 B channel model (Medbo and Schramm, 1998). For a fair comparison, we adjusted the codebook size and the group size for both algorithms (see Table III). For comparison we plot the quantized CBF with $K = 1$ (no subcarrier grouping) and 4-bit codebook. We observe from the curves of 64 feedforward bits, that the best performance is achieved with advanced clustering CBF with $K = 8$ and 8 bit codebook, and the worst performance is observed for the case of no subcarrier grouping ($K = 1$) and 1 bit codebook. The maximum achieved sum rate by employing advanced clustering is 21.8% higher than the achieved sum rate by employing no subcarrier grouping with 1 bit codebook.

Table III. Settings for 64 bits of feedforward

Method	Group Size	Number of Groups	Codebook Size
Quantized CBF	1	64	2
Clustering	2	32	4
and	4	16	16
Advanced clustering	8	8	256

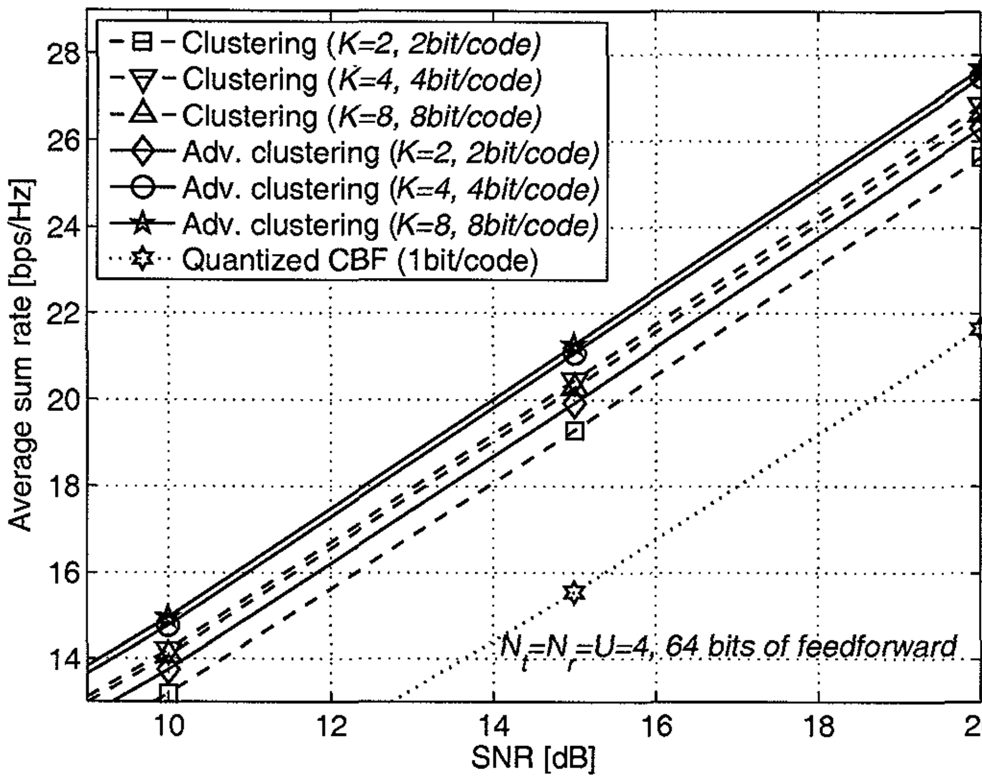

Figure 21. Sum rate as a function of the SNR in CBF for MU-MIMO-OFDM systems.

V.4.2.2 Quantized CBF for Different Channel Profiles

Figure 22 shows the sum rate versus the root-mean-square (RMS) delay spread for fixed dimensions ($N_t = N_r = U = 4$) and fixed number of feedforward bits per user. We consider the HiperLan/2 channel models A, B, C, and E (Medbo and Schramm, 1998) to show the performance of the proposed algorithms under different channel conditions. It is observed that advanced clustering has better sum rate performance for all the channel profiles; cluster size 8 gives better performance with a RMS delay spread less than or equal to 150 ns. Advanced clustering performs better than all proposed methods in this chapter, as can be seen by comparing figures 20 and 22.

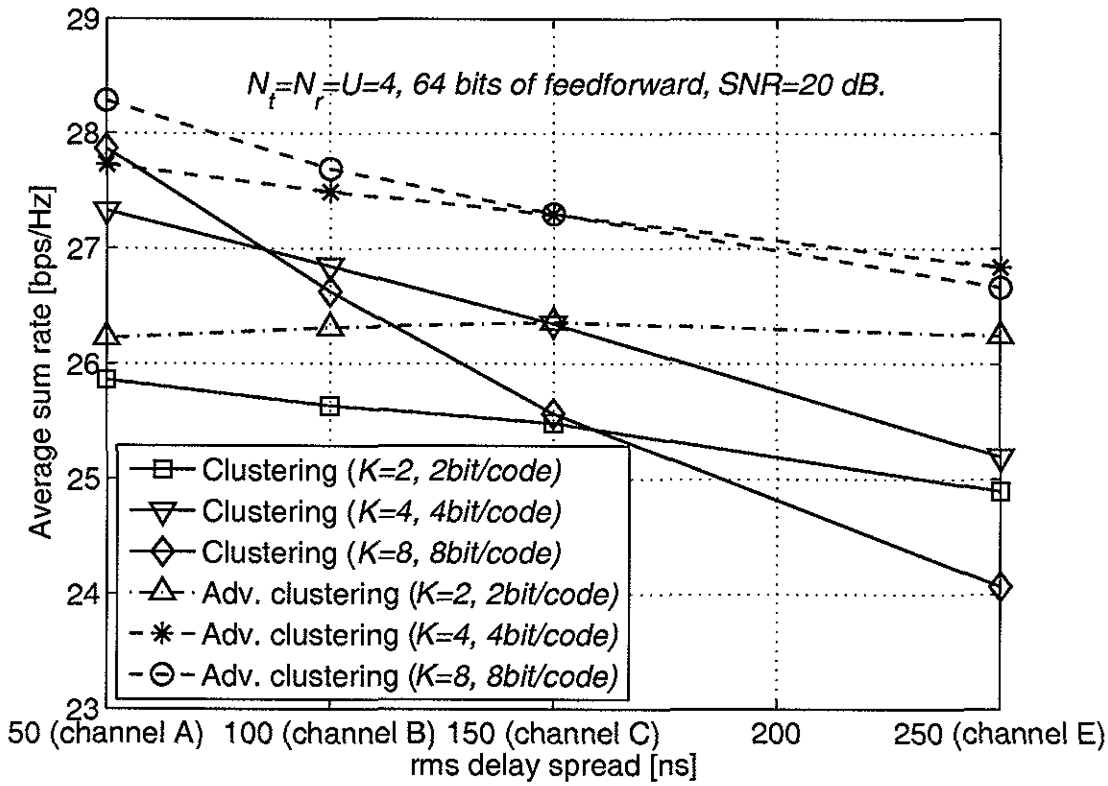

Figure 22. Sum rates as a function of the channel delay spread.

V.4.2.3 Quantized CBF with Path Loss Effect

In order to show the performance in a more realistic environment, we examine the achieved sum rate when the MSs are located at different distances from the BS. We adopt the path loss model described in (Erceg and et al., 2004) for a channel with 100 ns of RMS delay spread

(Hiperlan/2 channel model B). The noise power is assumed to be equal to -117 dBm per sub-carrier and we set the transmit power equal to -7dBm per carrier for each user. Figure 23 shows the sum rate versus distance for advanced clustering, clustering, and quantized CFB without subcarrier grouping, we assumed that the users are located at the same distance from the BS. The performance for advanced clustering, with the users uniformly distributed in a range from 1 to 100 meters, is included for comparison purposes. As in previous subsections, advanced clustering performs better than clustering.

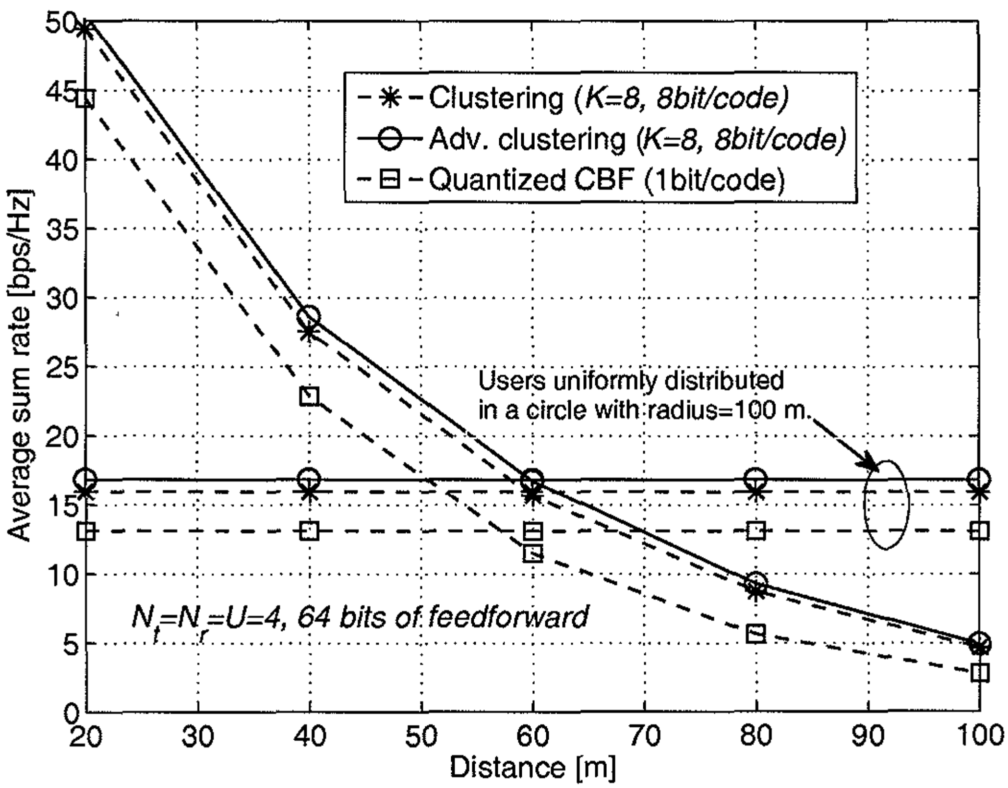

Figure 23. Sum rates as a function of the distance.

V.4.3 Conclusions

In this chapter, four approaches for choosing the best quantized beamforming vector that represents a subcarrier group, in the downlink of a coordinated beamforming MU-MIMO-OFDM system were presented.

The proposed algorithms offer good feedforward overhead reduction under zero interference and maximum SINR constraints.

By comparing the obtained results in subsection V.4.1 and subsection V.4.2, we can conclude that the algorithm with the best performance is advanced clustering. We observed that for 20 dB of SINR in the $N_t = N_r = U = 4$ configuration, clustering achieves a feedforward reduction up to 75% (from 256 to 64 bits) with reduction in sum rate of just 2.15%, while for getting the same savings in feedforward bits, interpolated CBF gives a reduction of 3.36% in sum rate. We observed that adaptive clustering performs better than clustering and interpolated CBF under different delay spread conditions.

From the subsection V.4.2 we can conclude that the proposed algorithm that achieves the best sum rate performance is advanced clustering. Advanced clustering selects the subcarrier that maximizes the sum rate of the system and performs better than all algorithms presented in this chapter under different delay spread conditions.

Chapter VI

Conclusions and Future Work

Reaching the end of this thesis, a brief summary of the main contributions and findings of the dissertation are given in this chapter. Some suggestions on future research directions are discussed with a brief summary of the possible extensions to this work.

VI.1 Conclusions

The dissertation was focused on the downlink of MU-MIMO and MU-MIMO-OFDM wireless systems, in which multiple-antennas are employed at both the transmitter (base station) and the receivers (mobile stations) to provide high sum rates through a jointly calculation of the transmit beamforming and receive combining vectors.

In the first part (chapters II and III), we considered MIMO flat fading channels and proposed three non-iterative and one iterative algorithms to implement coordinated beamforming.

The proposed non-iterative algorithms were based on applying a perfect interference cancellation at the transmitter before calculating the quantized beamformers. Improved joint receive quantization and suboptimal quantization algorithms are able to achieve higher sum rates than similar algorithms found in the literature. The low complexity algorithm reduces the feedforward overhead by a factor of $(U-1)/U$ and is able to achieve almost the same sum rate as joint receive quantization, when $N_r = N_t = U = 4$ and the number of feedforward bits is greater than or equal to two.

Our iterative proposal is the optimization of the existing iterative CBF algorithm based on singular value decomposition. It improves the CBF-SVD algorithm in terms of BER performance and reduces the number of iterations in the order of 50% for $N_r = N_t = K = 4$.

In the last part (chapters IV and V) we considered MIMO frequency selective channels, where OFDM is applied to convert the wideband channel in a set of flat fading subchannels. We exploited the correlation between subcarriers for reducing the feedforward overhead in the low rate control link of the system. Four algorithms were proposed, all of them offer a good feedforward reduction and were optimized under maximization sum rate criteria. The algorithm that achieves the highest sum rate is advanced clustering, where a subcodebook is generated in order to obtain a feedforward overhead reduction under zero interference and maximum sum rate constraints.

VI.2 Future Work

In this section, we provide a few directions for future research related to this dissertation.

Our proposals were designed by assuming perfect channel estimation at the transmitter and receiver. A good extension of this work would be to analyze the performance by considering a more practical scenario that includes errors due to channel estimation and delay in the feedforward link.

An interesting option to continue our research is to propose new algorithms for feedforward overhead reduction by considering the case where the user channels are spatially or temporary correlated. The same considerations can be assumed for the subcarriers in a MU-MIMO-OFDM system in order to reduce the overhead in the feedforward link.

It is possible to reduce the number of iterations of the algorithms CBF-SVD and CBF based on matrix inversion by a suitable user selection method that guarantees the algorithms' convergence after few iterations. User selection algorithms can also be studied to reduce the feedforward overhead by selecting users with quantized beamformers that can be represented with fewer bits.

The non-iterative algorithms described in chapter II were proposed after CBF for MU-MIMO-OFDM proposals, so an interesting research opportunity is to study extensions of these algorithms to MU-MIMO-OFDM.

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Appendix A

Abbreviations

BC	Broadcast Channel
BS	Base Station
BER	Bit Error Rate
CBF	Coordinated Beamforming
CBF-SVD	CBF based on Singular Value Decomposition
CP	Cyclic Prefix
CSI	Channel State Information
DPC	Dirty Paper Coding
DFT	Discrete Fourier Transform
FDD	Frequency-Division Duplex
IBIQ	Iteration-Based Independent Quantization
IDFT	Inverse Discrete Fourier Transform
i.i.d.	independent and identically distributed
JRQ	Joint Receive Quantization
LTE	Long Term Evolution
MAC	Multiple-Access Channel

MIMO	Multiple-Input Multiple-Output
MMSE	Minimum Mean Square Error
MRC	Maximum Ratio Combining
MS	Mobile Station
MU-MIMO	Multiuser MIMO
OFDM	Orthogonal Frequency Division Multiplexing
RMS	Root Mean Square
SDMA	Space Division Multiple Access
SINR	Signal-to-Interference-plus-Noise-Ratio
SVD	Singular Value Decomposition
TDD	Time-Division Duplex
WiMAX	Worldwide Interoperability for Microwave Access
ZF	Zero Forcing