

1 **Velocities in the Analysis of cGPS Data from Volcanoes, Somma-Vesuvius Volcano,**  
2 **Italy.**

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15  
16 **Abstract**

17 GPS/GNSS monitoring of surface deformation in volcanoes is a valuable tool for studying  
18 ongoing magmatic processes on active volcanoes, but displacements measured at different  
19 sites do not have a common reference level related to a known deformation status of the  
20 volcanic edifice. Velocities do not have this disadvantage, since they all have a well-defined  
21 reference zero level, so that positive or negative velocities measured on a given station  
22 component mean that the station is moving towards or away from the direction of the  
23 components, respectively. We propose that velocities, obtained directly from cGPS  
24 displacement measurements, may be useful to interpret different aspects of the observed

25 volcano deformations, and we present five tools that can be useful in visualizing and  
26 comparing measurements at different stations. The proposed technique and tools are  
27 illustrated by applying them to a public and complete dataset of continuous GPS time series  
28 of the Somma-Vesuvius Volcano, Italy.

29

30 **Keywords:** GPS volcano monitoring, Somma-Vesuvius volcano, velocities, Volcanic  
31 hazard.

32

### 33 **Introduction**

34 Whether obtained exclusively from the Global Positioning System (GPS) or incorporating  
35 other constellations of the Global Navigation Satellite System (GNSS), the continuous  
36 GPS/GNSS (cGPS/cGNSS) observations are an important tool for monitoring of volcanoes  
37 and early warning systems (Dixon *et al.* 1997; Lowry *et al.* 2001; Bartel *et al.* 2003; Janssen  
38 *et al.* 2002; Cabral-Cano *et al.* 2008; Moran *et al.* 2008; Lee *et al.* 2015; Larson *et al.* 2010;  
39 Savage *et al.* 2010; Ávila-Barrientos *et al.* 2021). These observations are used to characterize  
40 the activity associated with volcanoes, and the cGNSS data analyses show a correlation  
41 between surface deformation and magma migration (Mogi 1958; Nishimura *et al.* 2001;  
42 Mann *et al.* 2002; Miyagi *et al.* 2004; Fournier *et al.* 2009; Hotta *et al.* 2019). Continuous  
43 observations of ground deformation can contribute to reduce volcanic risk, particularly for  
44 volcanoes sited near populated areas (Tilling 2008).

45 Signals originating in the volcanic upper conduit, which may include seismicity and  
46 ground deformation at different scales, can be used as an indicator of eruptive activity in  
47 volcanoes due to increasing pressure in magma reservoirs (Chaussard *et al.* 2013, Ghosh *et*  
48 *al.* 2021). Ground deformation changes of the volcanic edifice precede the eruptions by times

49 that range from hours to months (Lee *et al.* 2015; Janssen 2007), and these changes could be  
50 applicable for timely evacuation (Salzer *et al.* 2014).

51 The results of cGNSS monitoring at various sites, or stations, located on or near a  
52 volcanic edifice, are time series corresponding to the three components of position, usually  
53 referred to as displacement, at each site, and the values of each of these series are measured  
54 with respect to reference values that, in general, vary from one site to another, and that have  
55 no relation to a given state of deformation of the volcanic edifice, since it is impossible to  
56 know exactly what is the state of repose of a given volcano.

57 To correctly interpret the information from cGNSS monitoring, it is necessary to be  
58 able to compare the observations obtained at different stations; and many studies do  
59 qualitative comparisons but, without a common reference level, quantitative comparison is  
60 not possible. Here, We present a set of tools for visualizing and comparing measurements at  
61 different stations applied to a public dataset of continuous GPS time series of the Somma-  
62 Vesuvius Volcano, Italy.

63

#### 64 **The Somma-Vesuvius Volcano data**

65 Each feature of the approach we propose will be illustrated by application to the Somma-  
66 Vesuvius Volcano data set of cGPS displacements used by De Martino *et al.* (2021), who  
67 kindly made the data freely accessible. De Martino *et al.* (2021) present an introduction to  
68 the volcano itself and to the data set. Here, we will review only some aspects of the post-  
69 processing of the displacement time series and concentrate on the application of the velocity  
70 approach to gather information about the volcanic processes.

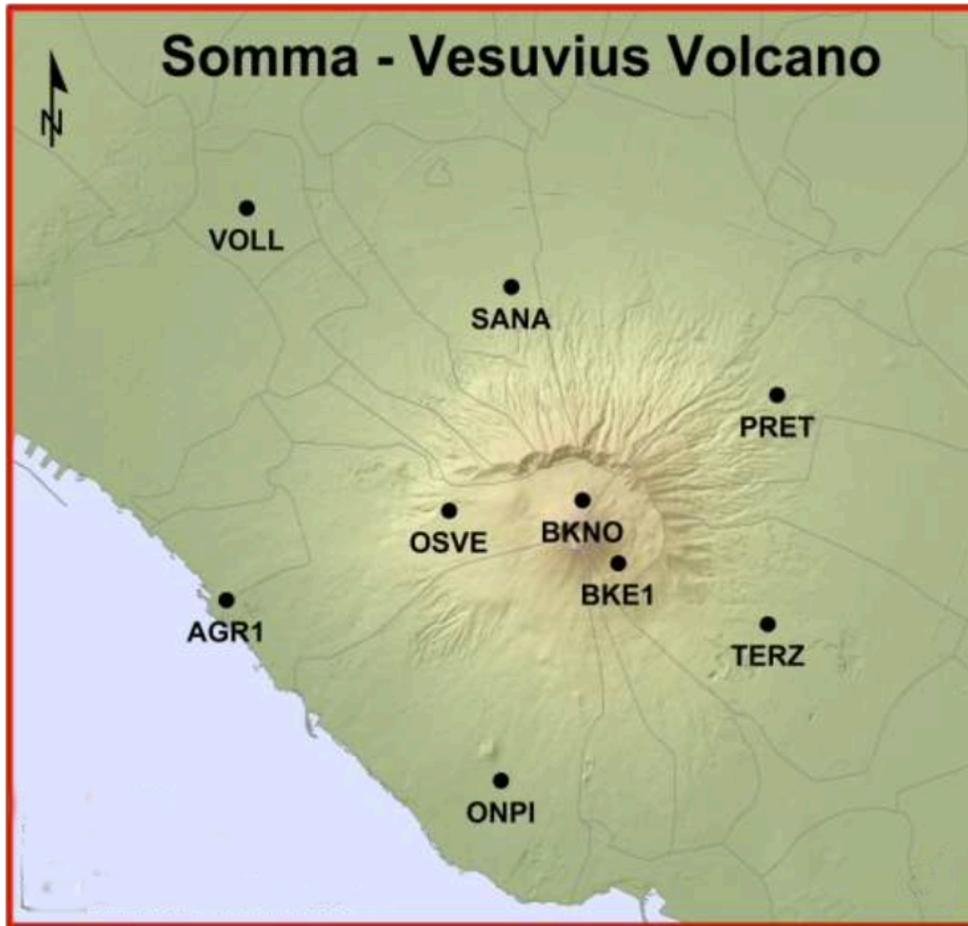
71           Nine cGPS stations monitor the activity of the Somma-Vesuvius Volcano ([Table 1](#)  
72 and [Fig. 1](#)), for details on the instrumentation the reader is referred to [De Martino \*et al.\* \(2021\)](#)  
73 and references therein.

74

75           **Table 1** cGPS stations located on the Somma-Vesuvius Volcano.

<b>Station</b>	<b>Longitude (°)</b>	<b>Latitude (°)</b>	<b>Elevation (km)</b>
AGR1	14.343	40.811	0.070
BKE1	14.439	40.819	0.864
BKNO	14.430	40.830	0.959
ONPI	14.411	40.779	0.123
OSVE	14.397	40.828	0.624
PRET	14.477	40.849	0.209
SANA	14.412	40.869	0.156
TERZ	14.475	40.808	0.180
VOLL	14.348	40.883	0.033

76



77

78 **Fig. 1** cGPS stations located on the Somma-Vesuvius Volcano (modified from [De Martino](#)

79

*et al.* (2021).

80

81 The original final daily position time series are shown in [Fig. 2](#), where time is

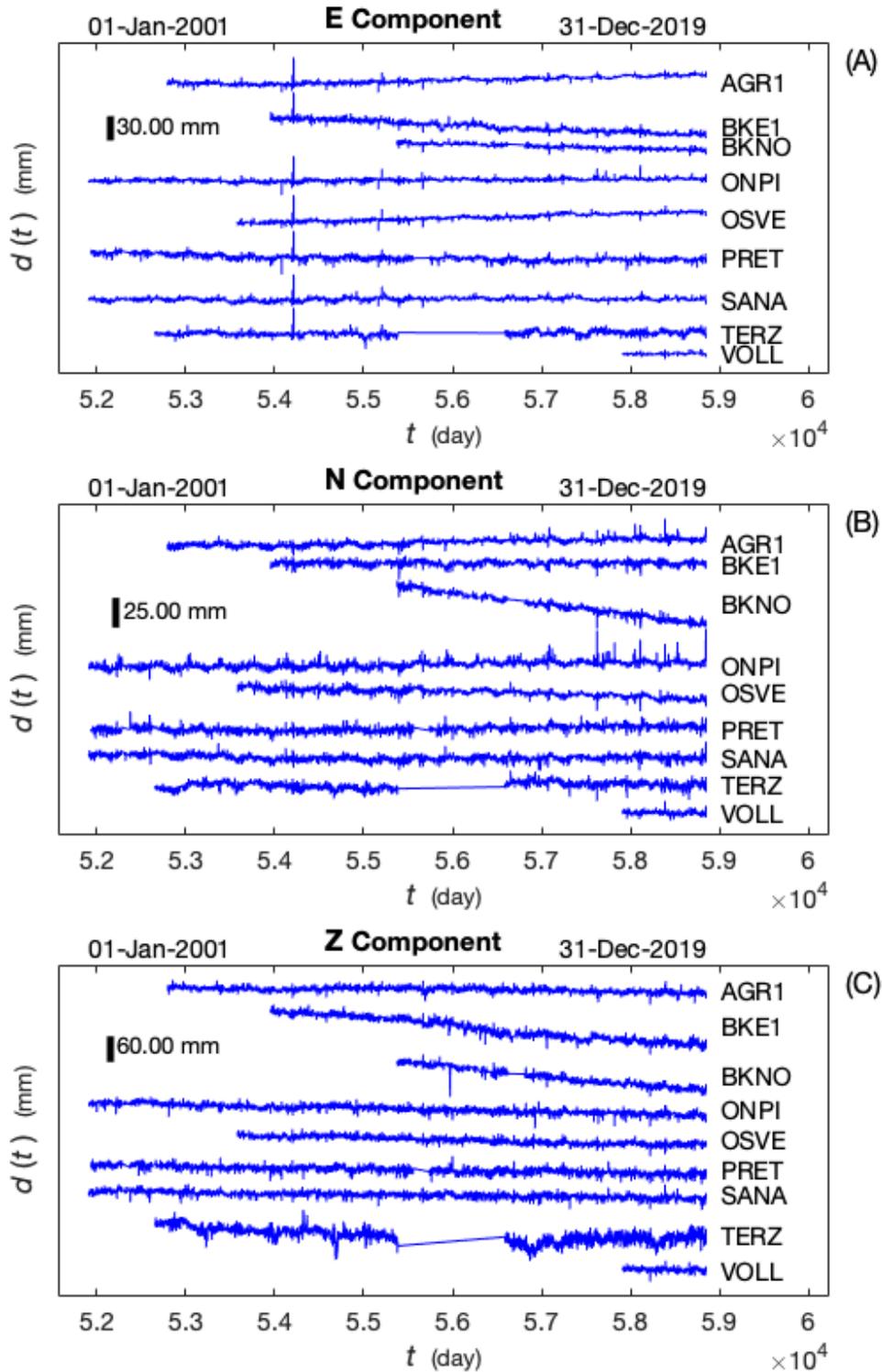
82 expressed in modified Julian days ([De Martino et al. 2021](#)). The series show strong high-

83 frequency noise that obscures the intermediate- and long-period behavior, and they have

84 gaps, i.e. missing data. Since we need equispaced data, interpolation is necessary, but large

85 gaps cannot be interpolated without introducing unwarranted information, so we will use

86 only segments having gaps  $\leq 3$  data.



87

88 **Fig. 2** Original final daily position displacement time series obtained from [De Martino et al.](#)

89 [\(2021\)](#) E-W component (A), Nort-South component (B), and Z or U component (C).

90

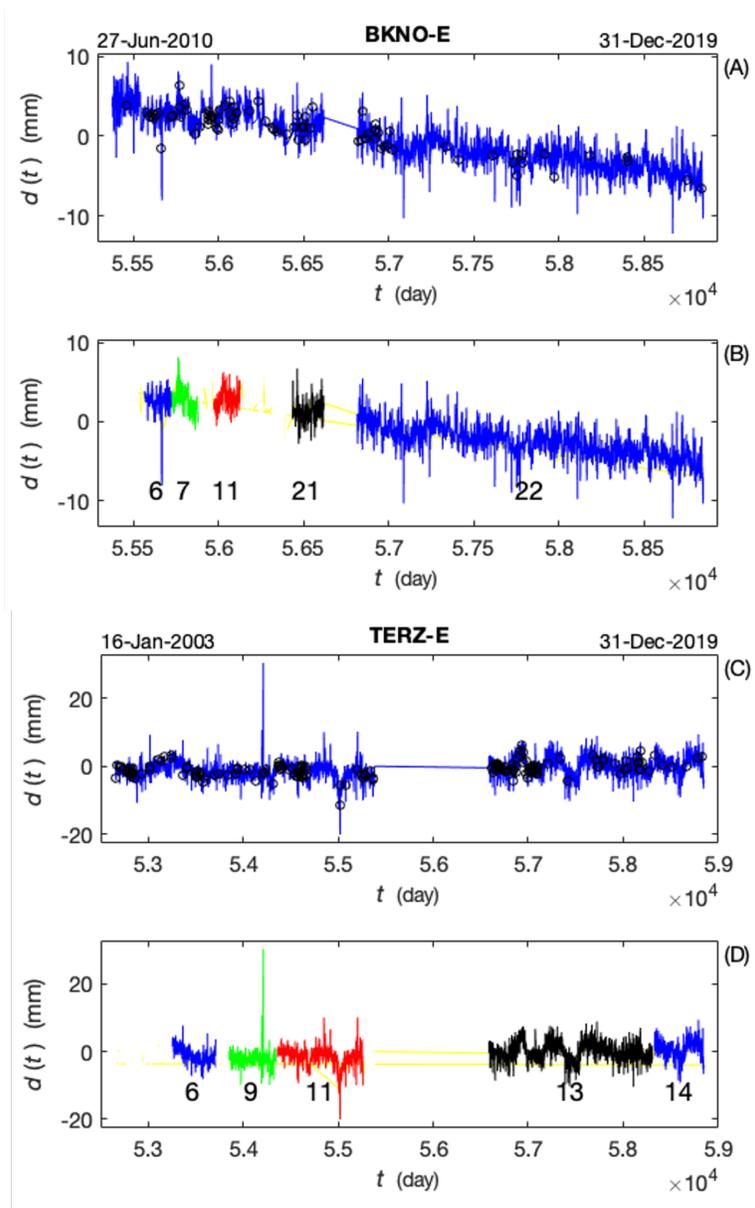
91           We tried several interpolation techniques by randomly creating gaps in a long series,  
92 interpolating the missing values, and comparing the spectrum of the reconstituted series with  
93 that of the original one. We found, surprisingly, that the interpolation that caused the smallest  
94 changes in the spectrum was linear interpolation, so this was the interpolation method used  
95 for the series shown below.

96

97           Figure 3 is an example of the interpolation and segmentation process for two stations.  
98 The original series are shown in (A) and (C), where circles indicate interpolated values. Since  
99 long-period features in the series show durations of about 100 samples and since the filtering  
100 we will need (discussed below) introduces tails about 70 samples long, we will only keep  
101 segments larger than 300 samples. These segments, identified by their occurrence order, are  
102 the ones used for further processing.

103           Since the displacement series have large high-frequency noise and since we will be  
104 using velocities derived from the displacement time series and differentiation enhances the  
105 high frequencies, it was necessary to low-pass the signals; we did this by means of a  
106 Butterworth filter (Hamming 1977) with reference frequency  $f_c = 0.01 \text{ day}^{-1}$  and filter order  
107  $N_B = 6$ . Other filter settings were tried, but we found that the above mentioned settings were  
108 the ones which gave sufficiently good results without over-filtering.

109



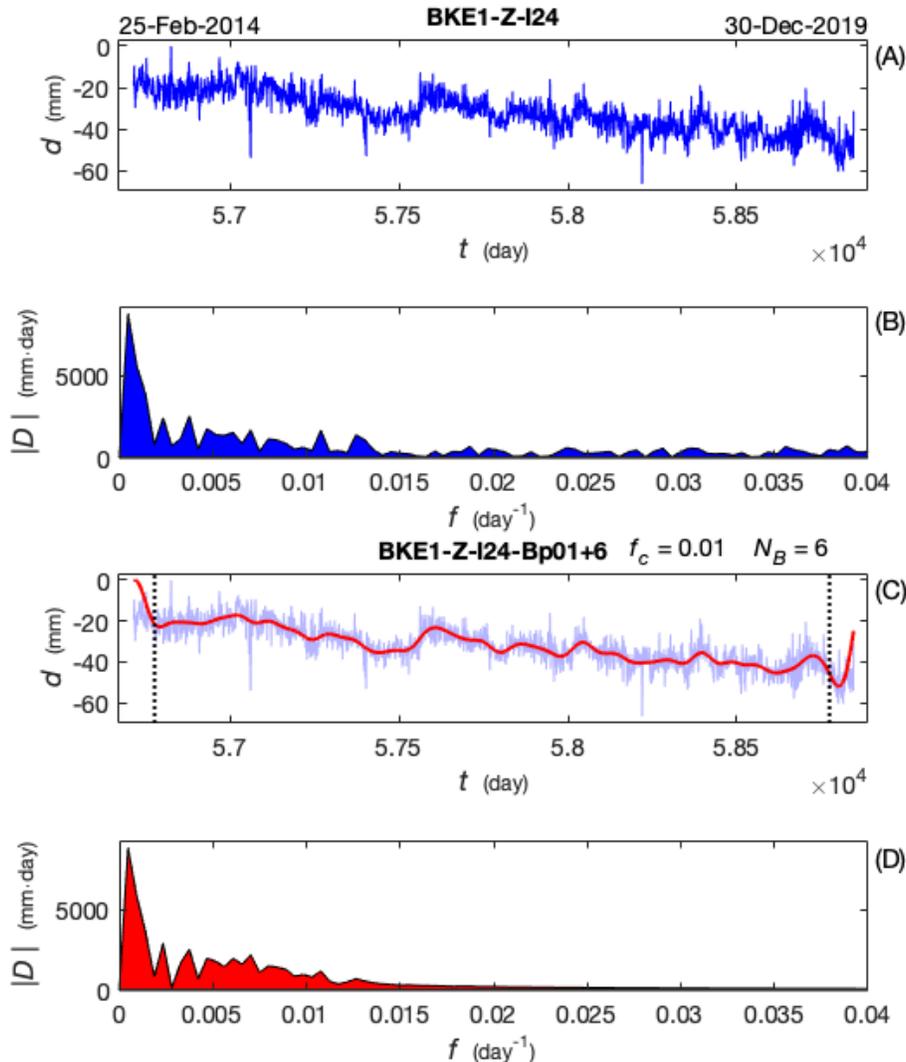
110  
 111 **Fig. 3** Example of the interpolation and segmentation process. (A) shows the original series  
 112 BKNO-E (East-West component), and the circles indicate the interpolated values, and (B)  
 113 shows the continuous series that are saved for further use. (C) and (D) are the corresponding  
 114 series for TERZ-E.

115

116 [Figure 4](#) shows an example of the low-pass filtering of the displacement time series:  
 117 the original (equispaced) series is shown in (A), and its Fourier amplitude spectrum is shown

118 in (B). The filtered series is shown in (C) as a thick red line over the original time series (light  
 119 blue), and its amplitude spectrum is shown in (D).

120



121

122 **Fig. 4** Example of filtering. Original interpolated segment (A) and its Fourier amplitude

123 spectrum (B); the filtered series is shown in (C) as a thick red line over the original in light

124 blue, and its spectrum in (D). The dotted vertical lines in (C) indicate the tails from the

125 filtering to be discarded.

126

127 **Velocities**

128 Once the high-frequency noise has been filtered out from the displacement series, the task of  
129 interpretation begins, but interpretation is complicated by the fact that the displacements  
130 constituting the time series do not have a common reference level, each series is referred to  
131 some zero value, different for each station, that is completely arbitrary with respect to the  
132 state of expansion or contraction in the volcanic edifice. It is impossible to know the  
133 displacement value that corresponds to a “repose” state, i.e. neither expansion nor  
134 contraction, of the volcano.

135 Hence, we take resort to displacement velocities

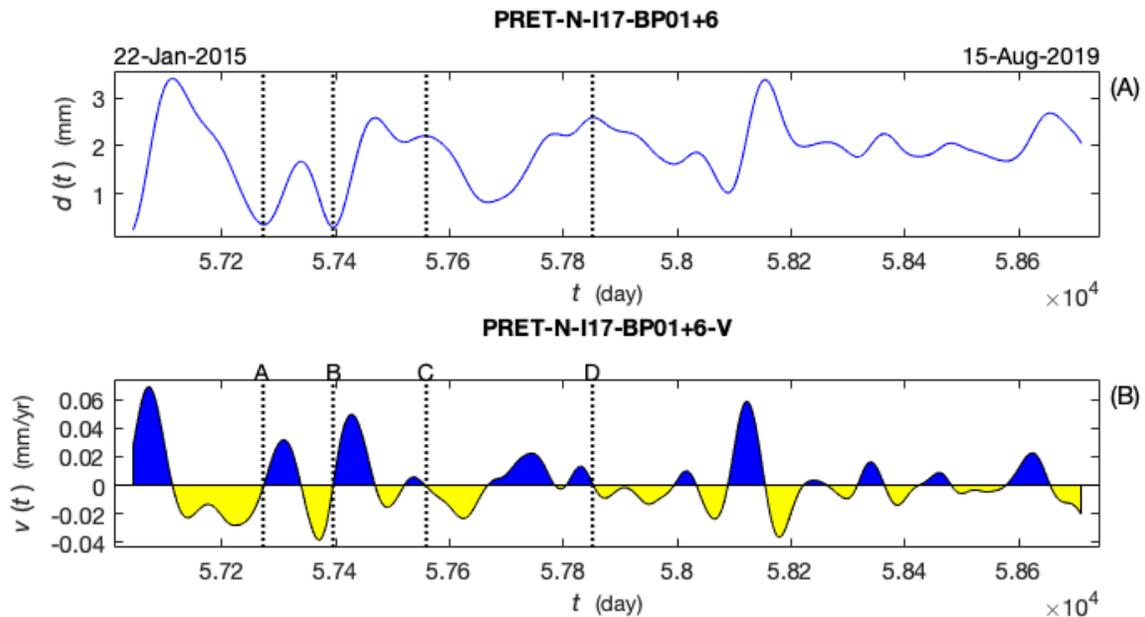
136 
$$v_i = (d_i - d_{i-1})/\Delta t, \quad (1)$$

137 where  $d_i$  is the  $i$ 'th element of the displacement series and  $\Delta t$  is the sampling interval.

138 Velocities have a well-defined zero level, and positive or negative velocities for a given  
139 component of a station indicate unequivocally that the station is moving in the positive or  
140 negative direction of that component, respectively, so periods of motion towards a given  
141 direction are easy to recognize and evaluate. Also, since zero levels are well defined, it is  
142 possible to construct 3D vectors that correspond to the total velocity at a given time for some  
143 station, which totally characterize how the station is moving in terms of total velocity  
144 magnitude (or amplitude), azimuth, and elevation.

145 [Figure 5](#) shows an example of a filtered displacement time series ([A](#)), where a  
146 reference zero level has been purposefully omitted, and the corresponding velocity time  
147 series ([B](#)), where positive and negative episodes have been assigned different shadings to  
148 make them easier to distinguish. The vertical dotted lines show how the zero velocity values  
149 correspond to the extrema of the displacement. This figure clearly illustrates the increase in  
150 frequency where one simple displacement excursion, such as the one going from [A](#) to [B](#)

151 (displacement goes simply up and down) corresponds to two velocity episodes, one positive  
 152 and one negative. A second, slightly more complicated, displacement excursion, going down  
 153 from C and coming back up at D, results in four velocity episodes, due to small details in the  
 154 displacement. Thus, since derivation is a high-pass filter, velocities show higher frequencies  
 155 and more details than the corresponding displacements.  
 156



157  
 158 **Fig. 5** Example of a filtered displacement time series (A) and the corresponding velocity time  
 159 series (B), where positive (blue) and negative (yellow) velocity episodes are shown with  
 160 different shadings to make them easier to distinguish. Vertical dotted lines with labels  
 161 indicate features discussed in the text.

162

### 163 **Interstation Velocity Differences**

164 Once the velocity time series have been obtained, it is useful to compare velocity records  
 165 from different stations, because the differences in velocity amplitudes and times may yield  
 166 valuable information about the deformation source. The most immediate way is to choose

167 segments of velocity time series sampled over the same period at different stations, let them  
168 be denoted by  $v_1$  and  $v_2$ , and plot them together in one or in adjacent figures, but many times  
169 it is difficult to see how differences at some time relate to those at other times. Another  
170 straightforward way is to plot the difference  $v_1 - v_2$ , but this is an ambiguous plot because  
171 the same value can be obtained from an infinite number of combinations of different values.

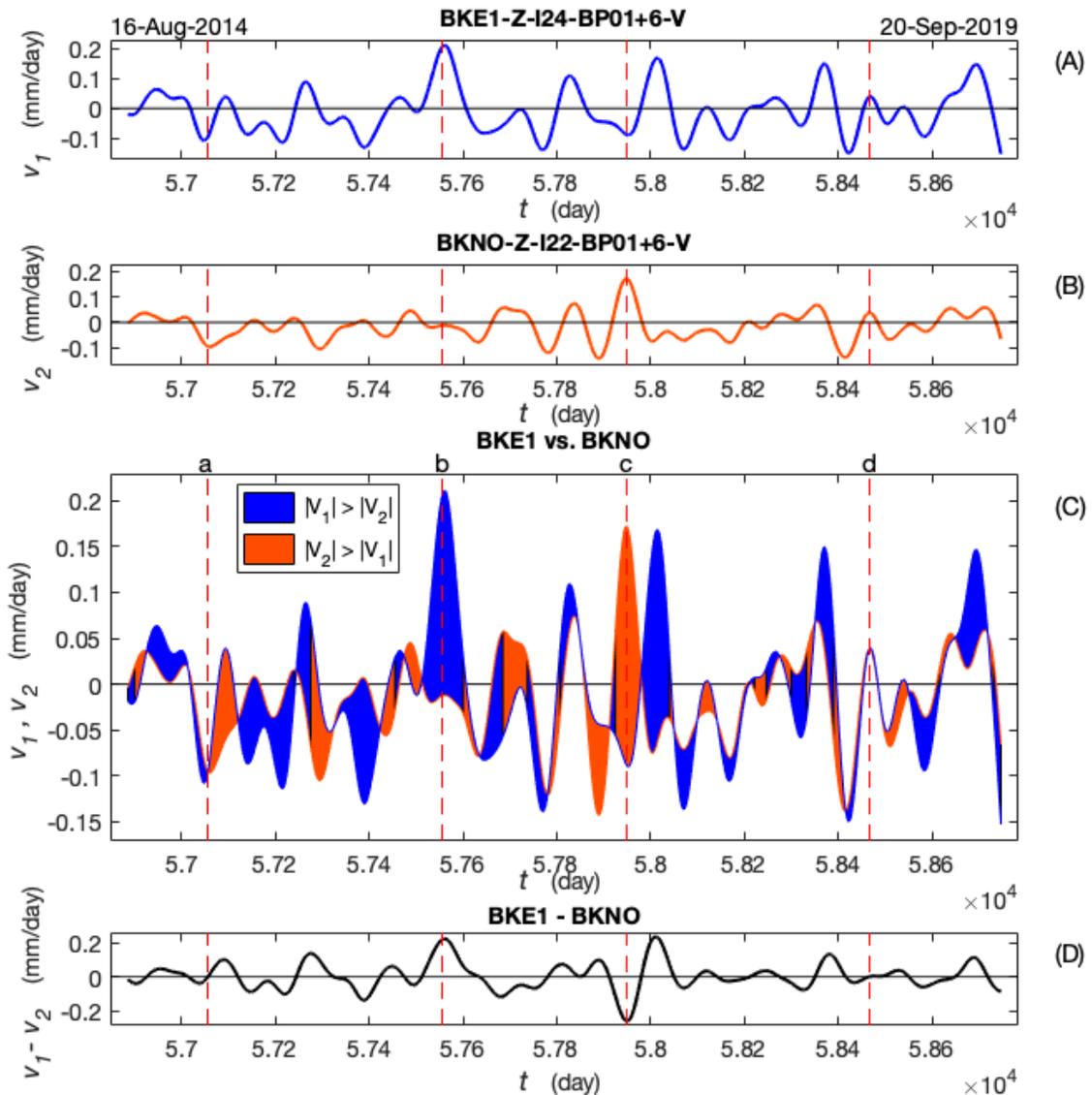
172 Here, we propose a way to present the data that, while not extracting additional  
173 information, makes the available information more accessible and easier to see and interpret.  
174 This method is shown in Fig. 6, which shows the series to be compared in panel (A), where  
175  $v_1$  is the vertical component from station BKE1, and panel (B), where  $v_2$  the vertical  
176 component from station BKNO, and their difference is plotted in panel (D). Panel (C) shows  
177 both series, but the differences between them have been shaded in two colors: blue indicates  
178  $|v_1| > |v_2|$  and orange corresponds to  $|v_2| > |v_1|$ , i.e. the colors identify which signal is  
179 dominant at the time. The resulting figure looks like a colored ribbon, where the width of the  
180 ribbon indicates the magnitude of the differences, and the upper and lower limits of the  
181 ribbon, together with the color, show at a glance both the sign and the amplitude of each  
182 component. This ribbon helps to interpret correctly the difference shown in panel (D), which,  
183 in turn, shows the numeric value of the difference corresponding to the ribbon's width.

184 It is possible to pick times, and optionally label them, to indicate features that help to  
185 understand the results. For instance, in Fig. 6, the time indicated by a vertical dashed line  
186 labeled c, corresponds to the maximum difference, chosen in panel (D) and corresponds to  
187 the maximum ribbon width in panel (C), when  $v_2$  had the largest (positive) amplitude (red)  
188 and  $v_1$  had a small amplitude that being negative contributed to the large difference. At time  
189 a, the width of the ribbon is almost zero (the difference between the signals is zero), as shown

190 in (D), but the height of the ribbon tells that the signals themselves are not zero, both are  
 191 negative and the amplitude of  $v_1$  is larger than that of  $v_2$  just before time a, and smaller just  
 192 after it. At time b, the ribbon is wide because  $v_1$  is large, while  $v_2$  is small. Finally, at time  
 193 d, the difference is again zero, but both signals are positive.

194 A feature symmetric with respect to zero, would indicate stations moving in the same  
 195 manner but in opposite directions for the observed component. For every chosen time, the  
 196 relevant values can be printed for later use.

197



198

199 **Fig. 6** Temporary comparison of velocity time series between A) BKE1 and B) BKNO  
200 stations for the vertical component. D) Difference between the signals,  $v_1 - v_2$ . C) Both  
201 signals, with shading between them colored according to which signal has the largest absolute  
202 value.

203

#### 204 **Co-velocities**

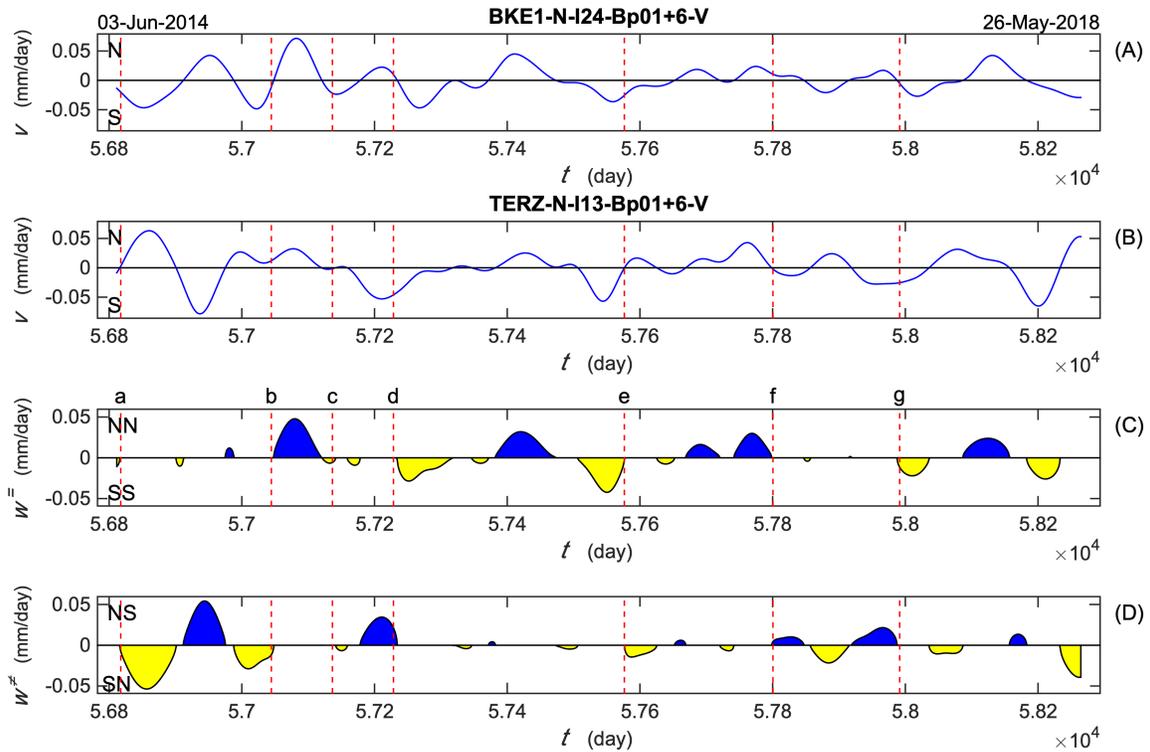
205 The second comparison between two stations is whether both stations are moving in the same  
206 or in opposite directions, evaluated by what we call *co-velocities* defined as

$$207 \quad w = \sqrt{v_1 v_2} \operatorname{sgn} v_1 = w^= + iw^\neq,$$

208 so that  $w$  is proportional to the product of the velocities and has units of velocity, it is real  
209 and equal to  $w^=$  when both velocities have the same sign, and imaginary and equal to  $iw^\neq$   
210 when they have different signs, and is positive or negative according to the sign of  $v_1$ ; if one  
211 or both velocities are zero, then  $w = 0$  because the question of whether both stations are  
212 moving in the same direction is meaningless.

213 An example of this comparison is shown in [Fig. 7](#), where the velocity series to be  
214 compared, the N-S components at BKE1 and TERZ, are shown in panels (A) and (B),  $w^=$  is  
215 shown in panel (C), where episodes when both stations were moving North are blue-shaded  
216 and when both were moving South are yellow-shaded. Panel (D) shows  $w^\neq$  episodes when  
217 the first station was moving North while the second station was moving South (blue-shaded)  
218 or vice-versa (yellow-shaded). Letters in the extreme left of each plot indicated the direction  
219 of motion; for panels (C) and (D) the first letter corresponds to the first station and the second  
220 letter to the second station.

221



222  
 223 **Fig. 7** Co-velocities for comparison of the N-S velocity components shown in (A) and (B),  
 224 (C) shows  $w^{\bar{}}$  and (D) shows  $w^{\tilde{}}$ .

225

226 A large episode of motion in opposite directions, starting at time a with BKE1 moving  
 227 South and TERZ moving North, then reversing directions, then reversing directions again  
 228 and ending at time b. This episode was followed by one with both stations moving in the  
 229 same direction between times b and c, followed by another with opposite directions between  
 230 times c and d, and another long episode of mainly same direction between times d and e.  
 231 Another episode with (mostly) opposite directions occurs between times f and g.

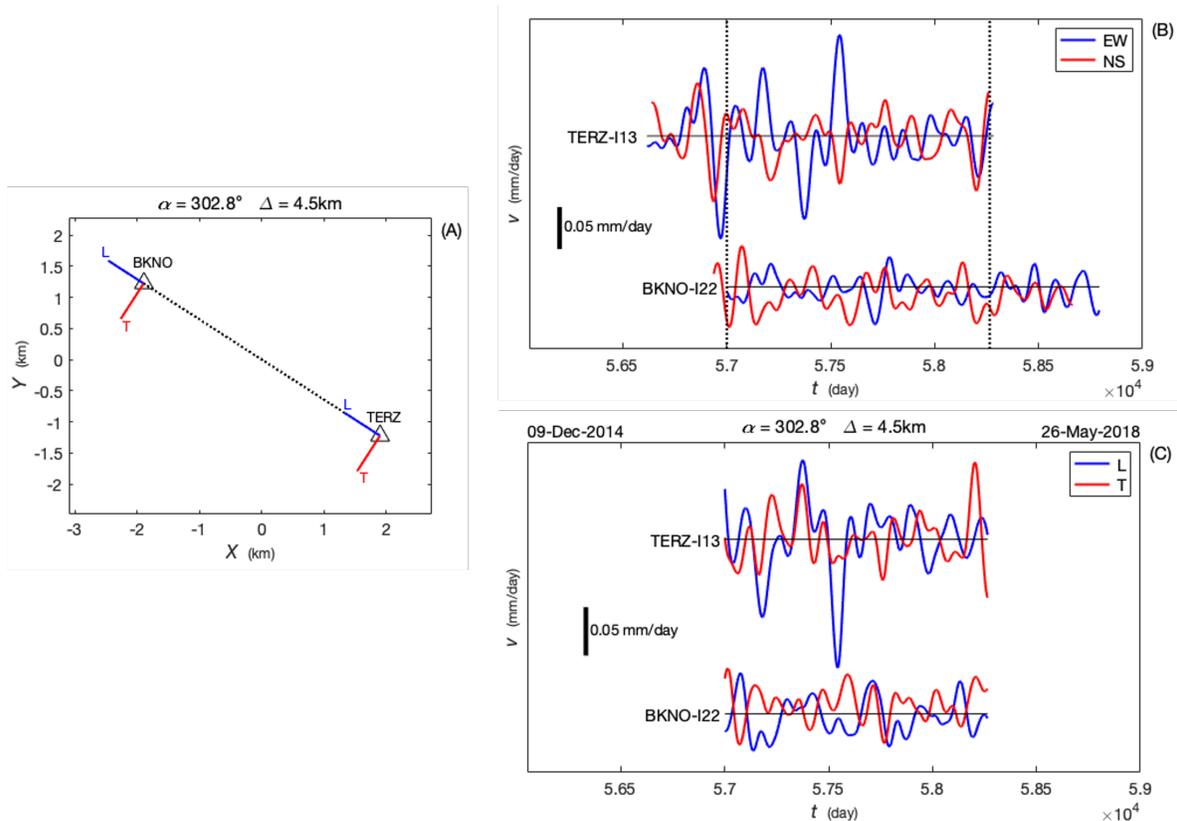
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### 233 Longitudinal and Transverse

234 The horizontal velocity components at two stations can be compared in the same way, but  
 235 their interpretation in terms of a possible source is not forthwith, because the location of the

236 source is unknown. Hence, as a third tool for easier interpretation, we propose rotating the  
 237 horizontal components to obtain components along the azimuth from the southernmost  
 238 station, henceforth referred to as station 1, to the northernmost one, station 2, which we will  
 239 call the longitudinal L, and components perpendicular to the azimuth direction, which we  
 240 will call transverse T. When the azimuth equals  $90^\circ$ , L coincides with E-W, and T with N-S.  
 241 These components, illustrated in Fig. 8, make interpretation more straightforward,  
 242 particularly the longitudinal one, because velocities with different signs mean that stations  
 243 are getting farther from or nearer to each other.

244



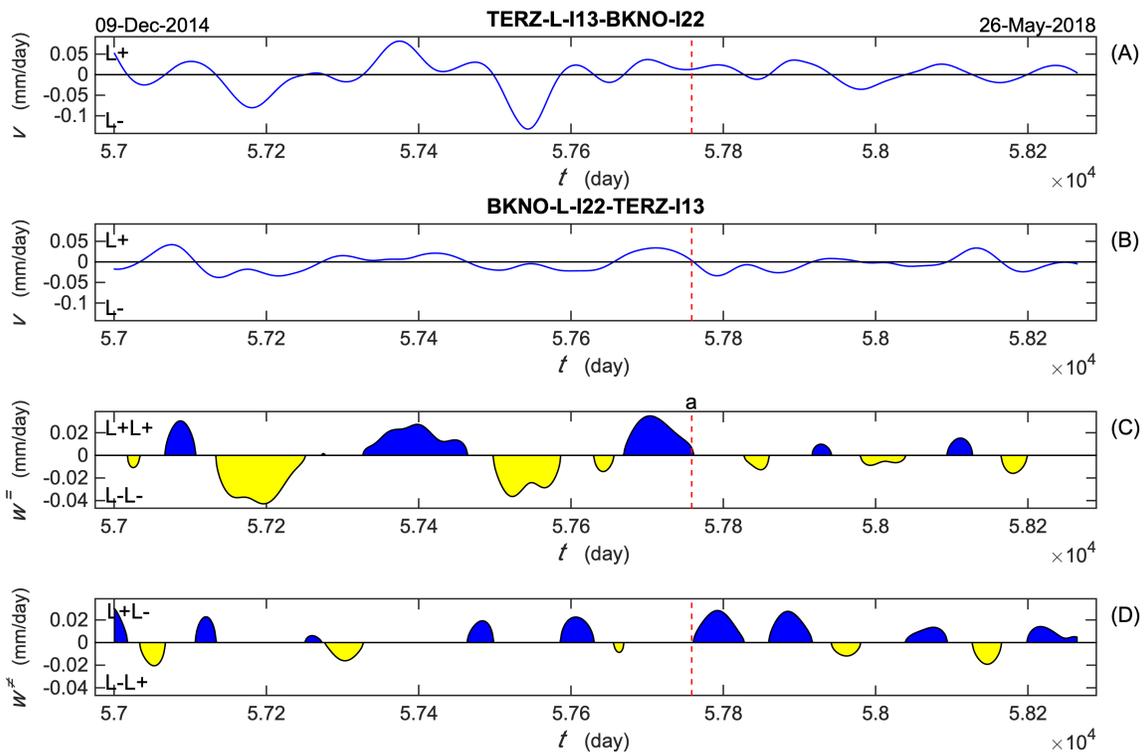
245 **Fig. 8** Longitudinal and transverse velocity components for two stations, in (A)  $\alpha$  is the  
 246 azimuth from station 1 (the southernmost one, TERZ) to station 2 (BKNO) and  $\Delta$  is the  
 247 distance between them; longitudinal components are represented as blue lines, and transverse  
 248 components as red lines. Panel B shows the E-W (blue) and N-S (red) components for both  
 249

250 stations; vertical dotted lines indicate the interval common to both stations. Panel C shows  
 251 the rotated longitudinal (blue) and transverse (red) components.

252

253 This comparison is particularly easy to make using co-velocities as shown in Fig. 9,  
 254 which shows that stations are moving mainly in the same direction until  $t = 57758$  days  
 255 (red vertical dotted line labeled a), indicated on the figure by a, and mainly in opposite  
 256 directions from then on.

257



258 **Fig. 9** Co-velocities comparison for longitudinal components at TERZ and BKNO. The  
 259 vertical dotted line indicates  $t = 57758$ .

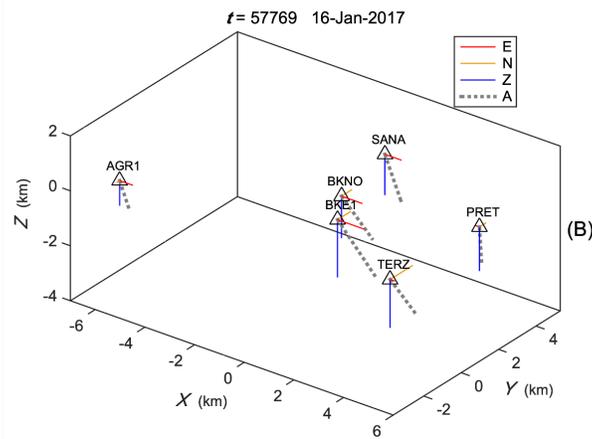
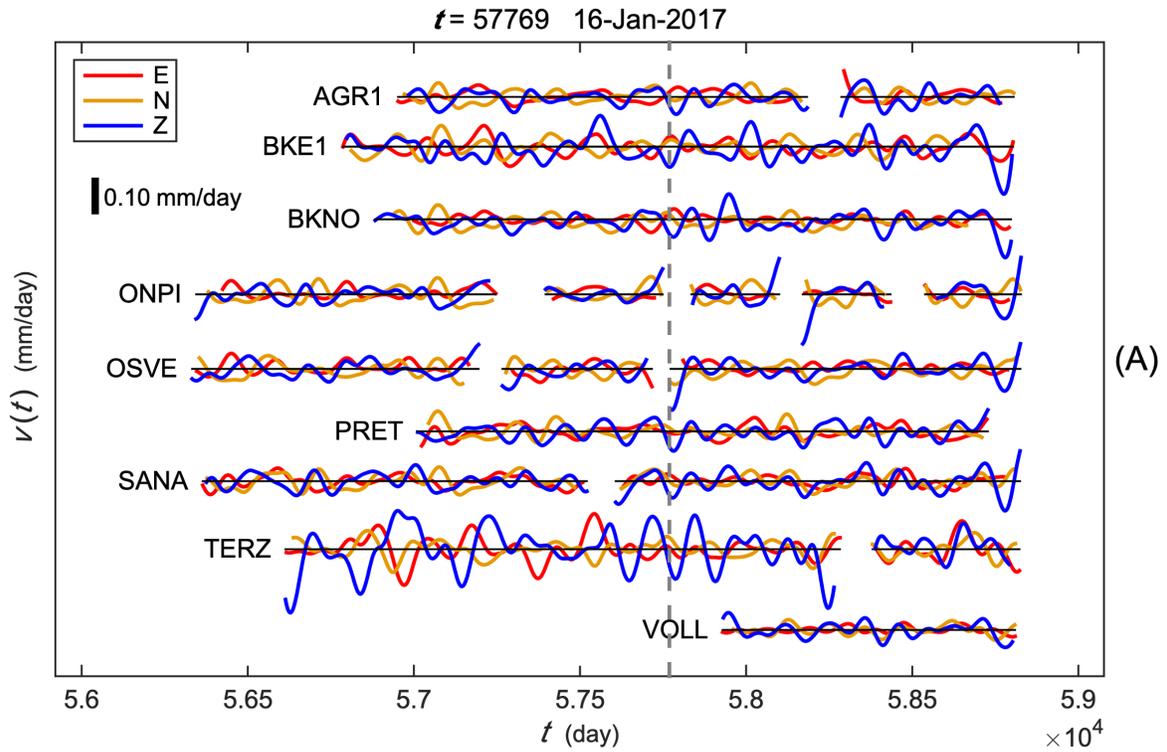
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261

## 262 Snapshots

263 The fourth interpretation tool considers together all velocities, shown as vectors, and their  
 264 vertical and horizontal components, through what we call snapshots. A snapshot is a record

265 of the velocity status of the network at a given time; consisting of both a listing of the  
266 measured velocity components (the Z or Up component is halved to correct for the free-  
267 surface effect) and the resulting velocity vector and its azimuth and elevation, and a 3-D  
268 figure that can be rotated to show how velocities were oriented at that particular time. [Figure](#)  
269 [10 \(A\)](#) shows the three velocity components at all stations, and the vertical dashed line  
270 indicates the time  $t = 57769$  of a snapshot. At this time velocities were as described in [Table](#)  
271 [2](#) and illustrated in [Fig. 10 \(B\)](#); stations ONPI, OSVE, and VOLL were not recording at the  
272 time.  
273



274

275 **Fig. 10** Display of all the velocity signals for the latter segments of the recorded series (A),

276 and a snapshot of velocities at all stations for time  $t = 57769$  (B).

277

278 **Table 2** Snapshot of velocities at time  $t = 57769$ , illustrated in [Fig.10 \(B\)](#)

$t = 57769$ (16 Jan., 2017)				
Station	$V_E$ mm/day	$V_N$ mm/day	$V_Z$ mm/day	$ V $ mm/day
AGR1	0.02	0.01	0.03	0.04
BKE1	0.01	0.02	0.01	0.02
BKNO	0.01	0.01	0.01	0.02
ONPI	0.01	0.01	0.01	0.02
OSVE	0.01	0.01	0.01	0.02
PRET	0.01	0.01	0.01	0.02
SANA	0.01	0.01	0.01	0.02
TERZ	0.01	0.01	0.01	0.02
VOLL	0.01	0.01	0.01	0.02

AGR1	0.0171	-0.0046	-0.0298	0.0347
BKE1	0.0353	0.0227	-0.0695	0.0812
BKNO	0.0274	0.0181	-0.0504	0.0601
PRET	-0.0055	0.0113	-0.0528	0.0543
SANA	0.0217	-0.0004	-0.0488	0.0534
TERZ	0.0058	0.0395	-0.0580	0.0704

279

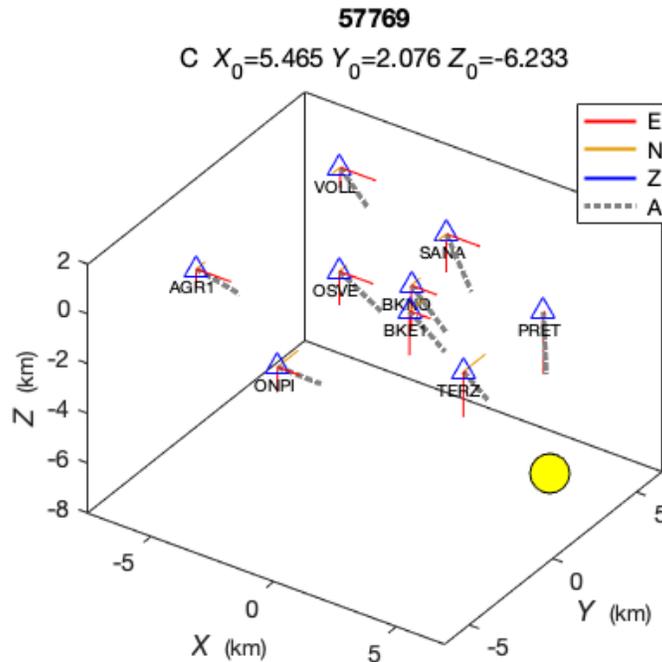
280 **Cartoons**

281 In order to interpret the velocity observations, it would be useful to have a realistic model of  
 282 the volcano and the source, but the source and its location are not known and the volcanic  
 283 structures are usually quite complicated and poorly known. Thus, to help interpret the  
 284 observations we take recourse to model so simplistic that we call it a cartoon, consisting of a  
 285 radially symmetric source in a homogeneous medium; velocity amplitudes decrease with  
 286 distance as the square root.

287 In such a model, an expanding source would cause stations on each side to register  
 288 velocities pointing away from the source, so that stations located on different sides of the  
 289 source would both be moving up but in opposite horizontal directions, while stations on the  
 290 same side would both be moving up and in roughly the same horizontal direction. For a  
 291 contracting source, stations on opposite sides of it would have velocities pointing towards  
 292 the source. Of course, more complicated source behaviors can give rise to motions that are  
 293 more difficult to interpret but knowing the relative motion directions might also be helpful.

294 Different source positions, contracting or expanding, can be proposed to see which  
 295 one results in velocities that resemble the observed ones.

296 [Figure 11](#) is a cartoon showing how the velocities would be for a source located at  
 297 (5.4645, 2.0763, -6.2332) (the source found below for an approximate location at time  $t =$   
 298 57769).



300  
 301 **Fig. 11** Cartoon showing how velocities would be for a contracting source represented by a  
 302 yellow circle. Velocity components are color-coded and the total velocity amplitude is  
 303 represented as a thick, gray dotted line, with lengths proportional to the velocity amplitudes.

304

305 The total velocity vectors are shown in Fig. 11 as dashed thick lines associated with  
 306 each station with lengths proportional to  $|V|$ , and their orientation points towards the source.

307

### 308 **Snapshots and Approximate Locations**

309 Snapshots very seldom show a simple pattern that could be easily interpreted using cartoons,  
 310 and many feature apparent contradictions that could be due to a complex distribution of the  
 311 source(s) or to site effects, but could also be caused by noise. Noise is unavoidable and can  
 312 be an important factor when velocities are small, in which case even a little noise can  
 313 significantly modify the velocity vector direction and can even change the polarity of a  
 314 velocity component.

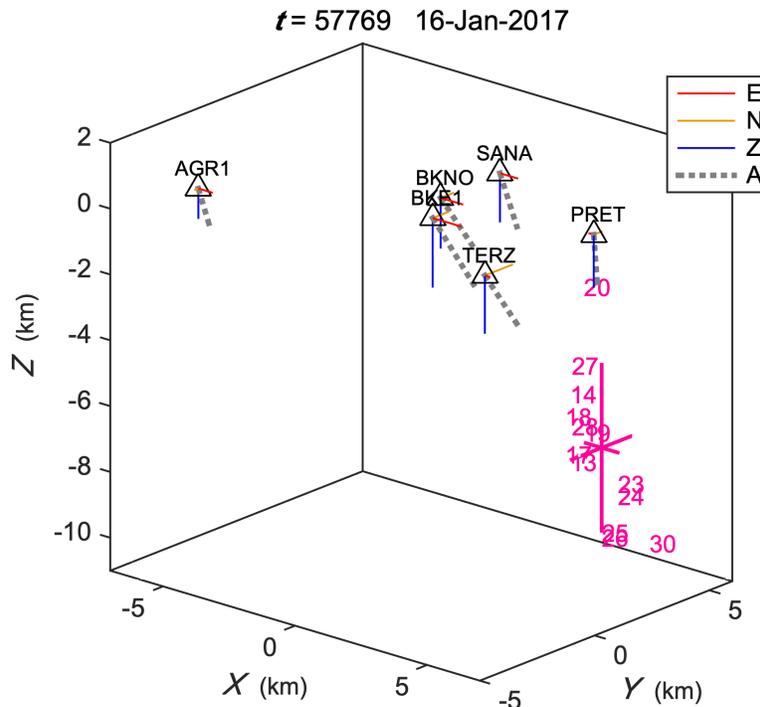
315           In rare instances snapshots show velocity vectors that agree reasonably well among  
316 them, and in these cases a very rough, approximate location can be tried. A way to obtain an  
317 approximate location is by propagating backwards the velocity vectors. For each pair of  
318 stations, the horizontal velocity components are propagated backwards to obtain their  
319 intersection in the X-Y plane, then the vectorial total velocity is propagated backwards from  
320 each station until it reaches the point below the horizontal intersection, usually at a different  
321 depth for each station. This procedure results in having generally two source solutions for  
322 each pair of stations, for a total  $N_s = N(N - 1)$  solutions, where  $N$  is the number of stations.

323           This method works perfectly for synthetic (cartoon) data without noise, but if noise  
324 is added to the synthetic data the solutions start to spread. For normal noise with a standard  
325 deviation as small as 0.05 times the largest velocity, some solutions can differ by large  
326 amounts.

327           Due to the above-mentioned discrepancies, solutions always differ for real snapshot  
328 data, but sometimes an acceptable, if approximate, location can be obtained by: first  
329 discarding stations featuring vertical velocities with different signs from those of the majority  
330 (if there is no majority or the number of positive vertical velocities is very close to that of  
331 negative ones, there is no reasonable solution). Next, assigning to each station a weight  
332 proportional to the total amplitude, and to each solution a weight corresponding to the product  
333 of the weights of the two stations involved. Finally, iteratively finding the weighted mean for  
334 each component of the source position, calculating the standard deviation of the solutions  
335 from that mean, and discarding outliers differing from the weighted mean by more than a  
336 given number of standard deviations or chosen by eye, until a satisfactory solution is found  
337 or until it is clear that there are no satisfactory solutions.

338 [Figure 12](#) shows one location obtained from the velocities at the time indicated by a  
339 vertical dotted line in [Fig. 10](#). From the 30 original solutions giving the raw source  
340 coordinates (with uncertainties corresponding to one standard deviation  $S$ )  $X_0 = -6.879 \pm$   
341  $30.308$  km,  $Y_0 = -5.780 \pm 19.296$  km, and  $Z_0 = -30.703 \pm 52.528$  km, after 3  
342 eliminations of outliers with positions  $>3S$  and 3 for  $>2.5S$  and manual elimination of 3  
343 solutions, from the remaining 13 solutions we get a final approximate location  $X_0 =$   
344  $-5.465 \pm 0.604$  km,  $Y_0 = 2.076 \pm 1.303$  km, and  $Z_0 = -6.233 \pm 2.522$  km. The  
345 velocities resulting in this location can be compared with the cartoon velocities for the same  
346 source location in [Fig. 11](#).

347



348 **Fig. 12** Example of approximate location from a snapshot of observed velocities. The  
349 numbers indicate the location of the solutions; there are two numbers for each pair of stations,  
350 assigned alphabetically. The weighted mean solution is at the intersection of the three  
351 magenta lines whose lengths correspond to the standard deviations in each direction.  
352

353

## 354 **Discussion and conclusions**

355       As mentioned above, using velocities has several advantages over using  
356 displacements, the main one is having a definite reference zero level, which lets velocities be  
357 compared and be treated as vectors. Since velocities are proportional to the instantaneous  
358 displacements (Eq. 1), any conclusions derived from velocities can be readily applied to  
359 displacements by going back to the displacement time series and taking into account the times  
360 or features of interest determined from the velocities.

361       However, working with velocities has some disadvantages. Since the derivative is a  
362 high-pass filter, the series require stronger low-pass filtering to give a useful picture of  
363 medium- and long-period features of the volcanic deformations. Also, velocities are sensitive  
364 to high-frequency noise that can distort information, particularly when velocities are small;  
365 yet small velocities can yield important information, a station moving towards or away from  
366 a source located directly East or West of it, should have null, or very small, N-S velocity  
367 components.

368       The approach and the tools we present here, while not contributing additional  
369 information from that contained in the position time series, can make some behaviors of the  
370 volcanic sources easier to appreciate, and comparisons easier to understand and evaluate.

371       We hope that the material presented in this paper, will be useful to scientists working  
372 with cGPS/GNSS data from volcanoes, and may help them to make contributions to the better  
373 understanding of the volcanic processes.

374

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380

381 **Statements and Declarations**

382 **Competing Interests**

383 The authors declare that there are no conflicts of interest or competing interests.

384

385 **Authorship contribution statement**

386 F. A. Nava: Conceptualization, Program code, Data analysis, Writing – First draft, review of  
387 subsequent drafts. L. Ávila-Barrientos: Data analysis, Writing – contributions to and review  
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389 all drafts, editing.

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394 **Data availability**

395 The data are available in De Martino et al. (2021)

396

397

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